APPLICATION OF LAPLACE TRANSFOM FOR SOLVING POPULATION GROWTH AND DECAY MODELS

BY

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DECLARATION

I, **NATUKUNDA PAMELLAH** declare that the content of this report is my original work and has never been published or submitted for any other degree award to any other university or higher institution of learning.

Signature--~~------ .. ---------- Date: 09/03/22

NATUKUNDA PAMELLAH

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APPROVAL

This project report was developed under my guidance and supervision.

Signature .... ~ Date:13/03/22

MR. TUMUHIMBISE BAZEYO EDSON

Research Supervisor

DEDICATION

I dedicate this book to my beloved parents Mr and Mrs Karuhize Misaach for their support towards my education.

AKNOWLEDGEMENT

First of all I thank God who enabled me to acquire my education and for protecting me up to this time, I will live to glorify and magnify his holy name

Much thanks goes to my parents for their encouragement, provision, support and wisdom they gave me since I was born, in my education up to this level.

Then I thank my supervisor Mr. Tumuhimbise Bazeyo Edson for his inspiring efforts in guiding me through the course of producing this report.

Then finally I would like to extend my appreciations to friends, coursemates and lecturers for the encouragement, love and knowledge they gave me.

May God bless you abundantly

ABSTRACT

This study is about application of Laplace Transfom for solving population growth and decay models. It is majorly concerened about on the linearity property of Laplace transform, How Laplace transform is used in solving population growth and decay model and How Laplace transform is used in solving population growth and decay models demonstrated

Laplace can be used to convert complex differential equations to a simpler form having polynomials, convert derivatives into multiple domain variables and then convert the polynomials back to the differential equation using Inverse Laplace transform, telecommunication field to send signals to both the sides of the medium. For example, when the signals are sent through the phone then they are first converted into a time-varying wave and then superimposed on the medium and for many engineering tasks such as Electrical Circuit Analysis, Digital Signal Processing, System Modelling.The given application show that the effectiveness of Laplace transform for solving population growth and decay problems. The proposed scheme can be applied for compound interest and heat conduction problems.I encourage mathematicians to use laplace transform model in solving differential equations and population models since it is perfect and easy to solve.

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CHAPTER ONE: INTRODUCTION

1.0 Introduction

This chapter comprise of the Background of the study, Statement of the problem, Objectives of the study, Scope of the study and Significance of the study

1.1 Background of the study

The population growth arise in the field of biology chemistry, physics, zoology and social sciences. Population growth is defined as the change in size of the population which can be either positive or negative over time depending on the balance of births or deaths.

The population growth (growth of a plant, a cell, an organ or a specie) is governed by the first order linear differential equation where

$$\frac{dN}{dT} \qquad KN \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

with initial conditions as

$$N() \qquad N_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where K is a positive real number. N is the amount of population at time t and IN is the initial population at time t.

Equation (1) is known as the Malthusian law of population growth.

Mathematically the decay problem is of the substance is defined by the first order linear ordinary differential equation

$$\frac{dN}{dT} \qquad -KN \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

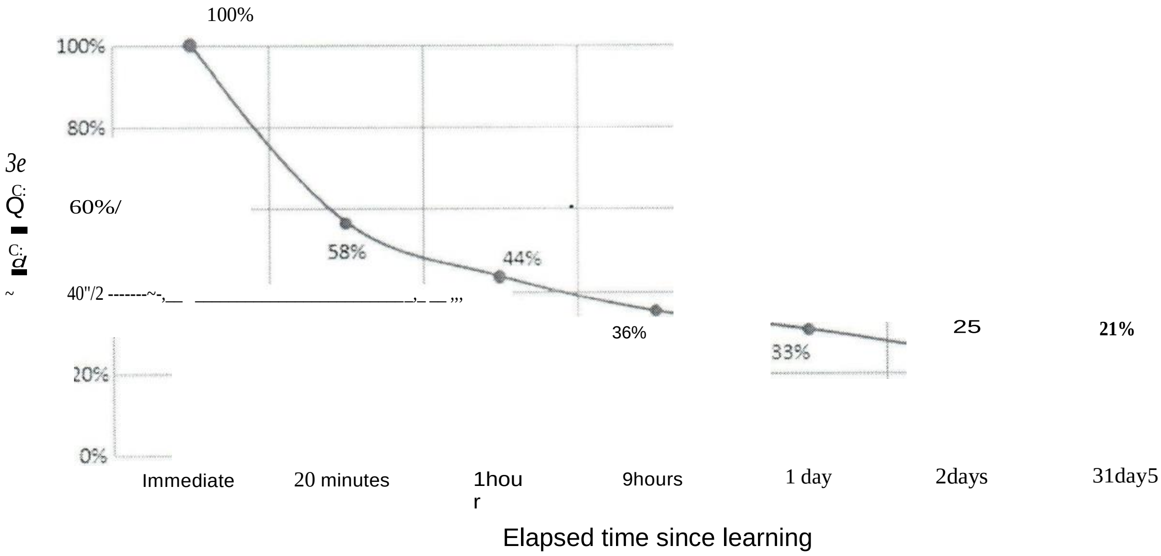
With initial conditions as

$$N_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Where N the amount of substance at time t, K is a positive real number and N is the initial amount of the substance at time t.

In equation (3) the negative sign in the right hand side (R.H.S) is taken because the mass of the substance is decreasing with time and so the derivative $\frac{dN}{dT}$ must be negative.

Laplace transform for decay problems



Decay is when numbers decrease rapidly in an exponential function so far every x-value on a graph, there is a smaller y-axis

In this section we represent Laplace transform for decay problem which is mathematically given by equation (3) and (4).

Applying Laplace transform on both sides of (3), we have

$$L\left\{\frac{dN}{dt}\right\} \qquad -Kl\{N(t)\} \text{ ....} (5)$$

Now Appling the property, Laplace transform of derivative of the function on (5) we have

$$sl\{N(t)\} - N(O) - Kl\{N(t)\} \qquad -KL\{N(t)\} \text{......................}(6)$$

Using (4) in (6) and on simplification, we have

$(s + K)\, L\{N(t)$ M

L{N(t)} $-- \dfrac{M}{(s + K)}$ ....(7)

Operating inverse Laplace transform on both sides of (7) we have

$N(t)$ $-1[''\}_{S+K}$

$N(t)$ ▬ $'1$

$N(t)$ Noe-kt .... (8)

Which is the amount of substance at time t.

Laplace transform was enunciated first English Oliver Heaviside (1850-1925) from operational methods while studying some electrical engineering problem.

Laplace transform takes a function of the time and transforms a function of a complex variable (s). Because the function is invertible, no information is lost and it is a reasonable nomenon. Each view has its uses and some features of phenomenon are easier to understand in the view of

the other.

We can use the Laplace transform to transform a linear invariant system from the time domain to the s- domain. This leads to the system function G((s) for the system. This is the same system used in the NY Quist criterion for stability. Laplace transform also transform analytic problems

to algebraic problems.

The Laplace transform of a function *f(t)* is defined by the integral;

Lf;(s) = *,er@oat*

For s=0 ,*f* is the complex value function of a complex number. tis called the

time variable in units (seconds)
We assume *f* contains no impulses at t=0

Laplace transform allows us to analyze,

Complicated circuits with sources.

Complicated systems with integrators, differentiations and gains.

Laplace transform has got the following properties,

Laplace transform is **linear;** if *a and b* are constants and *f and g* are functions then,

$$l(af + bg) = \qquad aLf) + bL(@)$$

A key property of Laplace transform is that; with some technical details, Laplace transform transforms derivatives int to multiplications bys

$L[f'(t);s] = SF(s)\ F(O)$ Valid for $Re(s) > a$

If f (t) has an exponential type a, then F(s) is an analytic function for $Re(s) > a$ and $F'(s) = -L(tf(@);s)$

A t-shirt rule, assume $f(t) = 0$, fort < 0 suppose $a > 0$then, $L[ft- as] = e\ @8F(s)$

1.2 Statement of the Problem

According to Sudhanshu et al (2018), many methods for solving population growth and decay models require much computation of work. In this project I found the solution of population growth and decay problems using Laplace transform for solving population and decay problems and some of the applications are given in order to demonstrate the effectiveness of Laplace transform for solving population growth and decay problems .

1.3 Objective

To find out the solutions of mathematical models that is population growth and decay models.

1.4 Specific Objectives of the Study

1. To establish the linearity property of Laplace transform

11. To find out how Laplace transform is used in solving population growth and decay model

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xiv. To demonstrate effectiveness of Laplace transform in solving population growth and decay models

1.5 Research Questions

1. What is the linearity property of Laplace transform
11. How is Laplace transform used in solving population growth and decay model
111. How is Laplace transform in solving population growth and decay models demonstrated

1.6 Significancy of the study

The study will be carried to find out the solutions of population growth and decay models without large computational work.

Laplace transform study will also help to find out the body of existing literature and knowledge in the field for further research in other subjects.

1. 7 Scope of the Study

The study will be confined to apply Laplace transform in solving population growth and decay models. It will be limited to find out whether it has any effect in solving population growth and decay models.

1.8 DEFITIONS OF TERMS

a. Laplace transform

This is defined by the integral $Lf;(s) =$ $f,e''rat$

b. Differential equations

These are partial differential equations (PDEs) if the unknown function depends on at least two variables.

c. A partial differential equation

The order of a differential equation is the highest order derivative that appears in the equation.

d. Population growth

This refers to rapid increase and decrease in the birth rate and death rates of the population. S xv.

e. Decay models

These refer to the process of reducing an amount by a consistent percentage rate over a period of time.

1.9 LIMIT A TIO NS OF THE MODEL

It should be noticed that our model is limited to the following;

Food, resources and a healthy environment are necessary but not sufficient conditions for growth. Even when resources are abundant, growth may be stopped by social factors. Of course, the society will not be suddenly surprised by the crisis point at which the area of land needed becomes greater than that available.

Symptoms of the crisis will begin to appear long before the crisis point is reached. Food prices will rise so high that some people will starve. Others will be forced to decrease the effective area of land they use and shift to lower quantity diets and thus caught in the death trap due to malnutrition.

There is a direct trade-off between producing more food and other goods needed by mankind. The demand for these goods is also increasing as population grows and therefore, the trade-off becomes continuously more apparent and more difficult to resolve

If the first priority is to produce food continued population growth and the law of increasing costs could rapidly drive the system to the point where all available resources were devoted to produce food, leaving no further possibility of expansion. The exponential growth of demand for food supply results directly from the positive feedback loop that is now determining the growth of human population

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the related literature about solving population growth and decay model using Laplace transform method and it will be guided by specific objectives which are; to establish the linearity property of Laplace transform, to find out how Laplace transform is used in solving population growth and decay model and to demonstrate effectiveness of Laplace transform in solving population growth and decay models

2.2 Linearity property of Laplace transform

if L{F(t)} = *f*(s) *and* L{G(t)} = *g(s) then*

L{aF (t)} + bG(st)} = *aL{F(t)}* + bL{G(t)}

if L{aF(t)} + *bG(t)* = *af(s)* + *bg(s) then* when

a,b are arbituary constant

example: find the laplace transform of 4t 3cos2t - se-t

L{At? 3cos2t + 5e} = 4L{t?} 3L{cos2t) + 5L(e)

2!)

- 4 (er}";+7 s 4 1

.GS,

8 3s 5

=−−−−+−s3 s?+4 s+1

The symbol L, which transforms F(t) into f (s), is often called the Laplace transformation operator. Because of the property oft expressed in this theorem, we say that L is a linear operator or that it has the linearity property (Spiehel,1965).

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2.2 How Laplace transform is used in solving population growth and decay model

The adjacent table shows Laplace transforms of various elementary functions. For details of evaluation using definition

S.N	F(t)	L{F(t)}=f(S)
1	l	1 – 5
2	T	1 s2
3	ɑ	2! s3
4	t",neN	n! n+1
5	t"n>-I	r(n + 1) n+1
6	ea	1 S- a
7	Sinat	a S?+ a?
8	Cosat	5 S? + a
9	Sinhat	a s?- a°
10	Coshat	5 S?- a?

2.3 Inverse Laplace transform

If L{F(t)}=f(s) then F(t) is called the Inverse Laplace transform of F(s) and mathematically it is defined by F(t)=L-[1] {f(s)} where Lis is the Inverse Laplace transform operator Raisinghania.M.D (2015)

2.4 Inverse Laplace transform of some elementary functions

According to Jeffrey A (2002), inverse Laplace transform of some elementary functions

S.N	L{F(t)}=f(S)	F(t)
1	1 5	1
2	1 52	T
3	2! s3	ə
4	n! n+1	tn,nEN
5	r(n + 1) n+1	tn,n>-1
6	1 S- a	ea'
7	a S?+ a	Sinat
8	5 S?+ a	Cosat
9	a S?- a	Sinhat
10	5 S? a	Coshat

2.5 Laplace Transform for Population Growth Model

In this section, we present Laplace transform for population growth problem given by **(1)** and (2).Applying the Laplace transform on both sides of (1), we have xix.

$L\{\frac{dN}{dt}\} = Kl\{N(t)\}$ ....(5)

Now applying the property, lap lace transform of derivative of function, on(S), we have

$sL\{N(t)\} - NO) = Kl\{N(t)\}$ ....(6)

Using (2) in (6) and on simplification, we have

$(s - K)L\{N(t)\} = N$

$L\{N(t)) = \frac{A}{(s-K)}$ ....(7)

Operating inverse Laplace transform on both sides of (7). we have N

$NO = L^{-1}\{\frac{1}{(s-K)}\}$

$NO = No$ ',-'

$N(t) = N_0eKt$ ....(8)

Which is the required amount of the population in terms oft.

Laplace transform for population Decay Model

In this section, we present Laplace transform for decay model which is mathematically given by (3) and (4).(Ali, et al 2019).

Applying the Laplace transform on both sides of (3). we have

$L\{\frac{dN}{dt}\} = -KN\{N(t)\}$ ....(9)

Now applying the property, Laplace transform of derivative of function, on (9), we have

$sl\{N(t)\} = KL\{N(t)\}$................. (10)

Using (4) in (10) and on simplification, we have

$(s + K)l\{N(t)\} = N$

$l\{N(t)\} = \frac{A}{(s+K)}$ ....(11)

Operating inverse Laplace transform on both sides of (11), we have

No-ref, S) Is + *K)*

$$N(t) = N_0 L^{-1}\{\frac{1}{(s + K)}\}$$

$$N(t) = N,e \text{ ....(12)}$$

Which is the required amount of substance at time t.

2.6 Effectiveness of Laplace transform in solving population growth and decay models

Application: The population of a city grows at a rate proportional to the number of people presently living in the city. If after two years, the population has doubled, and after three years the population is 20,000, estimate the number of people initially living in the city.

This problem can be written in mathematical form as:

$$\frac{dN(t)}{dt} = KN(t) \text{ .... (13)}$$

Where *N* denote the number of people living in the city at any time t and *K* is the constant of proportionality. Consider *NO* is the number of people initially living in the city at $t = 0$. Taking the Laplace transform on both sides of (13), we have

$$L\,r:1 = Kl\{N(t)\} \text{ .... (14)}$$

Now applying the property, Laplace transform of derivative of function, on (14), we have

$$sL\{N(t)\} - N(O) = Kl\{N(t)\} \text{ ....(15)}$$

Since at $t = 0$, $N = NO$, so using this in (15), we have

$$(s - K)L = N$$

$$L\{N(t)\} = \left(\frac{M}{s - K}\right) \text{ .... (16)}$$

Operating inverse Laplace transform on both sides of (16), we have N

$$N(DO) = L'\{ \quad "\} \quad (s-K)$$

$$1 \; N(t) = NL'\{-\} \quad (s-K)$$

$$N(t) = N_0 e^{Kt} \text{ ....(17)}$$

Now at= 2, $N = 2NO$, so using this in (17), we have

$$2N = Ne?$$

$$e? = 2$$

$$K = \frac{1}{2} \log_e 2 = 0.347 \text{ ....(18)}$$

Now using the condition at= 3, N = 20,000, in (17), we have

$$20000 = N_0 e^3 K \text{ ....(19)}$$

Putting the value of K from (18) in (19), we have

$$20000 = N,e"0.347$$

$$20000 = 2.832N_0$$

$$N = 7062 \text{ ....(20)}$$

Which are the required number of people initially living in the city.

Application: 2 A radioactive substance is known to decay at a rate proportional to the amount present. If initially there is 100 milligrams of the radioactive substance present and after two hours it is observed that the radioactive substance has lost 10 percent of its original mass, find the half-life of the radioactive substance.

This problem can be written in mathematical form as: $dN(t)$

$$a- \quad -KN(t) \text{ ....(21)}$$

Where N denote the amount of radioactive substance at time t and K is the constant of proportionality. Consider NO is the initial amount of the radioactive substance at time t = 0. Applying the Laplace transform on both sides of (21), we have

$$L\{\ldots\} = -KL\{N(t)\} \text{ .... (22)}$$

Now applying the property, Laplace transform of function, on (22), we have

$$sl[N(t)\} - N(0) = -KL\{N()\}$$ .... (23) Since at, $t=0$,

$N=N_0 = 100$, so using this in (23), we have

$$sL\{N(t)\} - 100 = -KL[N(t)\}$$

$$(s + K)L\{N(t)\} = 100$$

$$L[N(t)\} = \frac{100}{(s + K)} \text{ ....(24)}$$

Operating inverse Laplace transform on both sides of (24), we have 100

$$N(t) = L'\{\}_{(s + K)}$$

1

$= 100L '\{ = \}$ $(s + K)$

$$N(t) = 1ooe\text{-}Kt \text{ ....(25)}$$

Now at t=2, the radioactive substance has lost 10 percent of its original mass l 00mg so N= l

00-10=90, using this in (25), we have

$$90 = 100e\ 2i$$

$$e\text{-}2 - 0.90$$

$$K = -\frac{1}{2} \log_e 0.90 = 0.05268 \text{ ....(26)}$$

We required t when $N = \_Q = \frac{N}{2} = \frac{100}{2} = 50$ so from (25), we have

$$50 = 100e\text{-}Kt \text{ ....(27)}$$

Putting the value of K from (26) in (27). we have

$$50 = 100e\ 0.05268t$$

$0.05268t = 0.50$ 1

$$t = 0.052090.05$$

$$t = 13.157 \text{ hours } \text{........................ (28)}$$

Which is the required half-time of the radioactive substance.

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CHAPTER THREE
MATERIALS AND METHODS

3.1 Introduction

This Chapter covers the model formulation, assumptions and Model solutions. It looks at establishing the linearity property of Laplace transform, finding out how Laplace transform is used in solving population growth and decay model and demonstrating effectiveness of Laplace transform in solving population growth and decay models xxii.

CHAPTER FOUR

RESULTS

4.1 Laplace Model

$$Lf;(s)= \int_{0}^{\infty} e^{-st} f(t)dt$$

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Examples:

I) Let flt)= L, t>0

[I -A,| I
L{1}=F(s)=J· e- dt= limJ c- dt=lim--0= I.im .. --- .. -.=-.foralls>O
0 A->N U A··•"" S A-•oo S S S

2) Let ft)=t, t>0

[!tg=Ft)=]e 'tdt= lime'tdt= lim, st+ bl . . -e .
S A-,.~ {I }\--,..., S•

= lim [-;.- e-;s (sA + I)]= ¹ . for alls> 0
A-+ s7 sS

3) Let $f(t) = e^{at}$, $t \geq 0$

$$\mathcal{L}\{e^{at}\} = F(s) = \int_0^{\infty} e^{-st} e^{at} dt = \lim_{A \to \infty} \int_0^A e^{-(s-a)t} dt =$$

$$\lim_{A \to \infty} \frac{-e^{-(s-a)t}}{s-a}\Big|_0^A = \lim_{A \to \infty}\left[\frac{1}{s-a} - \frac{e^{-(s-a)A}}{s-a}\right] = \frac{1}{s-a}, \text{ for all } s > a.$$

4Let flt) = sn, at. t ?: 0

[I sin at; = H ~) = e -l>l sin at dt = hm . e sm at dt =
Λ-;.<i+ (I

hm - COS.d. A| ii - - f c · · cos al dt .. = - . . - - .. J.. c · CQS at dt =
Al a a'] a a'

! – - - - ·11·1 .... (· .... - e--.s-·t·-s.·.i_n_~_]] A . J· 1\ -l>l . d]=-" Fs
a A-+l a o + -~.- . c sm al a a
a" h

then.

$$F(s) = \frac{1}{a} - \frac{s^2}{a^2}F(s), \quad F(s)\left(\frac{a^2+s^2}{a^2}\right) = \frac{1}{a}, \quad F(s) = \frac{a}{s^2+a^2}, \; s \geq 0$$

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4.2 Model Application

In this section, some applications are given in order to demonstrate the effectiveness of Laplace transform for solving population growth and decay problems.

Application 1

The population of a city grows at a rate proportional to the number of people presently living in the city. If after two years, the population has doubled, and after three years the population is 20,000, estimate the number of people initially living in the city.

This problem can be written in mathematical form as:

$$\frac{dN(t)}{dt} = KN(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Where N denote the number of people living in the city at any time t and K is the constant of proportionality. Consider NO is the number of people initially living in the city at $t = 0$. Taking the Laplace transform on both sides of (13), we have

a4}_=at>> .-.-..Oo

Now applying the property, Laplace transform of derivative of function, on (14), we have

$$sL\{N(t)\} - N(O) = KL\{N(t)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Since at $t = 0$, $N = NO$, so using this in (15). we have

$$(s - K)L = N$$

$$L\{N(t)\} = \frac{M}{(s - K)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Operating inverse Laplace transform on both sides of (16), we have N

$$NO = L^{-1}\frac{1}{(s-K)}$$

$$= NE ',\_W]^{-1}\ \underline{1}\ NO$$

$$N(t) = N_0 e^{Kt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Now at= 2, $N = 2NO$, so using this in (17), we have

$$2N = N_0 e^{2K}$$

$$e^{2K} = 2$$

$$K = \frac{1}{2}\log_e 2 = 0.347 \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Now using the condition at= 3, N = 20,000, in (17), we have

$$20000 = N_0 e^{3K} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Putting the value of K from (18) in (19), we have

$$20000 = N_0 e^{3 \cdot 0.347}$$

$$20000 = 2.832N$$

$$N = 7062 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Which are the required number of people initially living in the city.

Application: 2

A radioactive substance is known to decay at a rate proportional to the amount present. lf initially there is I 00 milligrams of the radioactive substance present and after two hours it is observed that the radioactive substance has lost 10 percent of its original mass, find the half-life of the radioactive substance.

This problem can be written in mathematical form as:

$$\frac{dN(t)}{dt} = -KN(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Where N denote the amount of radioactive substance at time t and K is the constant of proportionality. Consider NO is the initial amount of the radioactive substance at time t = 0. Applying the Laplace transform on both sides of (21), we have

$$L\left\{\frac{dN}{dt}\right\} = -KL\{N(t)\} \quad \ldots\ldots\ldots\ldots (22)$$

Now applying the property, Laplace transform of function, on (22), we have

sL{N(t)} - *N(0)* = -*KL{N(t)}* .... (23)

Since at, t=0, *N=N,* = 100, so using this in (23), we have

sL{N(t)}- 100 = -*KL{N(t)}*

(s + *K*)*L{N(t)}* = 100

L{N(t)} = ($\frac{100}{s+K}$) .... (24)

Operating inverse Laplace transform on both sides of (24), we have 100

N(t) = C¹{--}
(s + *K*)

1

= 1ooc¹{--} (s + *K*)

N(t) = *1ooe-Kt* ....(25)

Now at t=2, the radioactive substance has lost 10 percent of its original mass l00mg so

N=l00-10=90, using this in (25), we have

90= 100e ?K

e-?K 0.90

1

K = -2 *loge0.90* = 0.05268 ....(26)

d M 100 f

We require t when N=- $\frac{=}{2}$ - = 50 $\frac{}{2}$ so rom (25), we have

50 = *l00e-Kt* ....(27)

Putting the value of K from (26) in (27), we have

50= 100e 0.05268t

-0.05268t 0.50 1

t< 0.052*09,0.05*

t = *l3.l57hours* ....(28)

Which is the required half-time of the radioactive substance.

4.3 Linearity property of Laplace transform

if L{F(t)} = *f(s) and* L{G(t)} = *9(s) then*

$$L\{aF(t)\} + bG(st)\} = aL\{F(t)\} + bL\{G(t)\}$$

if $L\{aF(t)\} + bG(t) = af(s) + bg(s)$ *then* when a,b

are arbituary constant

example: find the laplace transform of $4t^2$ - 3cos2t - se-t

L{At'- 3cos2t + 5e'}= 4L{t}- 3L{cos 2t} + 5Le)

-«(%) +t,'' _,)+s',

- s^3 $s^2 + 4$ $s + 1$

$$= \frac{8}{s3} - \frac{3s}{s^2+4} + \frac{5}{s+1}$$

The symbol L, which transforms F(t) into f (s), is often called the Laplace transformation operator. Because of the property oft expressed in this theorem, we say that Lis a linear operator

or that it has the linearity property.

4.4 How Laplace transform is used in solving population growth and decay model

The adjacent table shows Laplace transforms of various elementary functions. For details of evaluation using

definition

S.N	F()	L{F(t) }=f(S)
1	I	$\frac{1}{5}$
2	T	1 s?
◌̣	te	2! s3

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4	--_m::'1\	n! +1
5	**t.n>-l**	r(n + 1) n+1
6	ea	1 *S*- *a*
7	Sinat	*a* S?+ a
8	Cosat	*5* S?+ a
9	Sin hat	*a* S?- a
10	Coshat	*5* S?- a

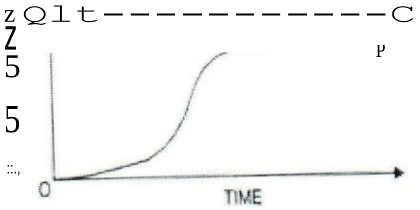
4.5 Demonstrating effectiveness of Laplace transform in solving population growth and decay models

Operation of the Model

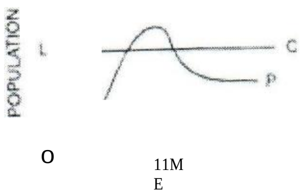
A major purpose in constructing the world model has been to determine which, if any, of these behaviour modes will be most characteristic of the world system as it reaches the limits to growth. The model shows four possible modes that a growing population can exhibit over time.

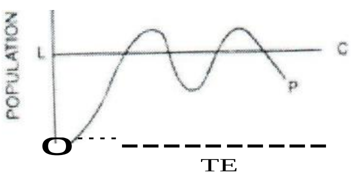
The mode actually observed in any specific case will depend on the characteristics of the carrying capacity. They are the level of population that could be sustained indefinitely by the prevailing physical and biological systems and on the nature of the growth process itself.

For example. a population growing in a limited environment can approach the ultimate carrying capacity of that environment in several possible ways. It can adjust smoothly to an equilibrium below the environmental limit by means of a gradual decrease in growth rate, as shown in Figure below where LC represents the carrying capacity of the world, while the OP curve represents the population growth curve.

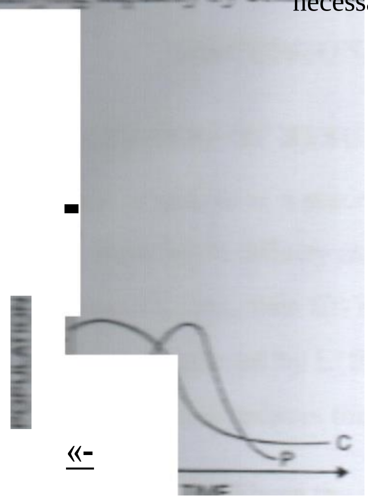


The second possibility is that it can overshoot the **limit** and then lie back either smooth or in an oscillatory way, as shown in Figures (B) and (C) respectively.





The last possibility is that can overshoot the limit and in the process decrease the ultimate ---_=**some** carrying capacity by con necessary nonrenewable resources. This is shown in Figure



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The most *careenistic* **fthe globe's** population and material outputs under different conditions have been shown in t hese figures. The purpose of the model is to identify the future policies that **er** . **an an** unstable lead to a stable rath behaviour mode.

CHAPTER FIVE

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

5.1 DISCUSSION OF RESULTS

imrion is **said to** be a piecewise continuous function if it has a finite number of breaks and it

does not blow **up to** infinity anywhere. Let us assume that the function ft) is a piecewise rims **function,** then ft) is defined using the Laplace transform. The Laplace transform of a **immiic is represented** by L{ft)} or F(s). Laplace transform helps to solve the differential

~=-re :t reduces the differential equation into an algebraic problem.

---'-----'-"-~ **--1-;,m:** Where the notation is clear, we will use an uppercase letter to indicate the

Laplace transf omm, forexample Lf; s) = F(s). The Laplace transform we defined is sometimes e-sided

called the on- Laplace transform. There is a two-sided version where the integral goes

from −∞ to ::r:..

Some of th - lz::b:-e rransforrnation properties are: sand[note: -

implies Laplace Transform]

In the inv erse **Laplace transform.** ♭.'e are provided with the transform F(s) and asked to find what

function w e **einitially. The** inverse transform of the function F(s) is given by:

f(t) = L⁻¹{F(s)}

For '--""......,-,,.,~- **=or the rwo** Laplace transform, say F(s) and G(s), the inverse Laplace transform is

defined by:

L⁻¹{aF(s)+bG(s) =aL'{F(s)}+bL: {Gs)}

Where a and b are constants.

..::>,c **be used to convert** complex differential equations to a simpler form having

polyno -- , **co::i\cr: deri,ariYes** into multiple domain variables and then convert the

olomnials back **to the differential** equation using Inverse Laplace transform,telecommunication field to send signals to both the sides of the medium. For example, when the signals are sent through the phone then they are first converted into a time-varying wave and then superimposed on the medium and for many engineering tasks such as Electrical Circuit Analysis, Digital Signal Processing, System Modelling

5.2 CONCLUSION

In this report, I have successfully developed the Laplace transform for solving population decay problems. The given application show that the effectiveness of Laplace transform for solving population growth and decay problems. The proposed scheme can be applied for compound interest and heat conduction problems

5.3 Recommendation

I encourage mathematicians to use laplace transform model in solving differential equations and population models since it is perfect and easy to solve.

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