

7.

ANALYSIS OF DIFFERENTIATION AND ITS APPLICATION

BY

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8.

DECLARATION

I Kirabo Flavia declare that this research report on the 'Analysis of Differentiation and its Application is entirely my original work and has not been submitted in any institution or University for the award of a Degree

Signature~. Date: 19/08/2019

APPROVAL

This report on the topic "Analysis of Differentiation and its Application" has been under my supervision and is now ready for submission to the Faculty of Education Kabale University for examination purposes.

Signature

A handwritten signature in blue ink, appearing to read 'Beegira B George', is written over a dotted line.

Mr. Beegira B George

Supervisor

Date: 24/08/2019

DEDICATION

This report is dedicated to all my Mathematics Teachers since primary level for the single bit of knowledge that I gained in Mathematics that kept on increasing until now when I am able to make my own proposal in one of the topics there.

Also, to my dear parents and relatives who instilled a spirit of learning, hard work and focus in me and continued to struggle hard to pay my school fees, may God bless you all.

ACKNOWLEDGEMENT

I sincerely acknowledge the great services rendered by many individuals in accomplishing this study.

Special thanks go to my supervisor Mr. Beegira B George for the technical guidance and support throughout the study. May God bless you abundantly.

I am so grateful to my dear parents and relatives for the financial support that enabled me go through this successfully. God bless you so much.

I thank friends especially Mr. Evarist Nshimyumana and classmates for the cooperation and contribution towards my research.

I also like to recommend the great work done by my lecturers, may the good Lord bless everyone accordingly for your contribution towards my academics.

TABLE OF CONTENTS

DECLARATION	ii
PROV AL	iii
DEDICATION	iv
ACKNOWLEDGEMENT	v
CHAPTER ONE	1
1.0 Introduction	1
1.1 Background to the study	1
1.2 Statement of the Problem	4
1.3 Purpose of the Study	4
1.4 Specific Objectives	4
1.5 Research Questions	4
1.6 Scope of the Study	4
1.7 Significance of the Study	5
1.8 Definition of Key Terms	5
CHAPTER TWO:	6
LITERATURE REVIEW	6
2.0 Introduction	6
2.1 Techniques of differentiation.....	6
2.1.1 The product rule	6
2.1.2 Quotient rule	6
2.1.3 Power rule	6
2.1.4 Sum rule	6
2.1.5 Chain rule	6
2.2. Applications of differentiation	7

2.3 Ancient Greek Precursors of the Calculus	7
CHAPTER THREE	10
~G:THODOLOGY	10
3.0 Introduction	10
3.1. Research Design	10
3.2 Study Area	10
3.2 Population of the study	10
3.2.1 Target population	10
3.2.2 Sample size	10
3.3 Sample Selection	11
3.3.1 Sample selection methods	11
3.5 Data collection methods and tools	11
3.5.] Questionnaire method.....	11
3.5.2 Docu1nentation	12
3.5.3 Interviewing	12
3.6 Research procedure	12
3. 7 Ethical considerations	12
3.10. Limitations of the Study	12
CHAPTER FOUR	14
DATA PRESENTATION, ANALYSIS AND INTERPRETATIONS OF FINDING	14
4.1 Introduction	14
4.2Back ground information of respondents.....	14
4.2.1 Gender of respondents	14
4.3 Findings on application of differentiation.....	15

4.4 Findings on the relationship between differentiation between velocity and acceleration in motion	15
4.5 Relationship between differentiation and calculating rate of change of chemical reactions	21
CHAPTER FIVE	25
SUMMARY OF FINDINGS, RECOMMENDATIONS AND CONCLUSIONS	25
5.1 Introduction	25
5.2 Summary of major findings	25
5.2.1 Application of differentiation	25
5.2.2 Relationship between differentiation between velocity and acceleration in motion	25
5.2.3 Relationship between differentiation and calculating rate of change of chemical reactions	26
5.3 Conclusion	26
5.4 Recommendation.....	27
5.5 Areas for further Research	27

CHAPTER ONE

1.0 Introduction

This study will focus on the analysis of differentiation and its application. This Chapter will cover the background of the study, statement of the problem, purpose of the study, objectives of the study, research questions, and scope of the study and the significance of the study.

1.1 Background to the study

Differentiation is a process of looking at the way a function changes from one point to another.

Given any function we may need to find out what it looks like when graphed. Differentiation tells us about the slope. As an introduction to differentiation we will first look at how the derivative of a function is found and see the connection between the derivative and the slope of the function.

Given the function $f(x)$, we are interested in finding an approximation of the slope of the function at a particular value of x . If we take two points on the graph of the function which are very close to each other and calculate the slope of the line joining them we will be approximating the slope of $f(x)$ between the two points. Our x -values are x and $x + h$, where h is some small number. The y -values corresponding to x and $x + h$ are $f(x)$ and $f(x + h)$. The slope m of the line between the two points is given by change in y over change in x

Where x and y are the two points.

Hence m is called the slope or change which is the differentiation.

The primary objects of study in differentiation are the derivative of a function, related notions such as the differential and their applications.

From the beginning of time man has been interested in the rate at which physical and nonphysical things change. Astronomers, physicists, chemists, engineers, business enterprises and industries strive to have accurate values of these parameters that change with time.

The mathematician therefore devotes their time to study the concepts of rate of change. Rate of change gave birth to an aspect of calculus known as differentiation.

There is another subject known as integration.

Integration and differentiation in broad sense together form subject called CALCULUS

Hence in a bid to give this research project an excellent work, which is of great utilitarian value the students in science and social science, the research project is divided into four chapters, each of these chapters broken up into sub units.

Historically, the primary motivation for the study of differentiation was the tangent line problem: -a given curve, find the slope of the straight line that is tangent to the curve at a given point. The word tangent comes from the Latin word "tangens", which means touching. Thus, to solve the tangent line problem, we need to find the slope of a line that is "touching" a given curve at a point, or, in modern language, that has the same slope.

In mathematics, **differential calculus (differentiation)** is a subfield of calculus concerned with the study of the rates at which quantities change. It is one of the two traditional divisions of calculus. the other being integral calculus (integration).

The primary objects of study in differential calculus are the derivative of a function, related notions such as the differential, and their applications. The derivative of a function at a chosen **input** value describes the rate of change of the function near that input value. The process of finding a derivative is called **differentiation**. Geometrically, the derivative at a point is the slope of the tangent line to the graph of the function at that point, provided that the derivative exists and is defined at that point. For a real-valued function of a single real variable, the derivative of a function at a point generally determines the best linear approximation to the function at that point.

Differentiation and integration are connected by the fundamental theorem of calculus, which states that differentiation is the reverse process to integration.

Differentiation has applications to nearly all quantitative disciplines. For example, in physics, the derivative of the displacement of a moving body with respect to time is the velocity of the body, and the derivative of velocity with respect to time is acceleration. The derivative of the momentum of a body equals the force applied to the body; rearranging this derivative statement

adds to the famous $F = ma$ equation associated with Newton's second law of motion. The reaction rate of a chemical reaction is a derivative. In operations research, derivatives determine the most efficient ways to transport materials and design factories.

Derivatives are frequently used to find the maxima and minima of a function. Equations **volving** derivatives are called differential equations and are fundamental in describing natural **phenomena**. Derivatives and their generalizations appear in many fields of mathematics, such as **complex** analysis, functional analysis, differential geometry, measure theory and abstract **-gebra**.

Application of differentiation in real life

~?\\Tecks occurred because the ship was not where the captain thought it should be. There was **01** a good enough understanding of how the Earth, stars and planets moved with respect to each **other**.

Calculus (differentiation and integration) was developed to improve this understanding.

Differentiation and integration can help us solve many types of real-world problems.

We use the derivative to determine the maximum and minimum values of particular functions e.g.cost, strength, amount of material used in a building, profit, loss, etc.).

Derivatives are met in many engineering and science problems, especially when modeling the behavior of moving objects.

Differentiation of Transcendental Functions, which shows how to find derivatives of sine, cosine, exponential and tangential functions.

Integration, which is actually the opposite of differentiation.

Differential Equations, which are a different type of integration problem, but still involve differentiation.

1. Tangents and Normal which are important in physics (eg forces on a car turning a corner)
2. Newton's Method - for those tricky equations that you cannot solve using algebra
3. Curvilinear Motion, which shows how to find velocity and acceleration of a body moving in a curve

- **Rate** - where 2 variables are changing over time, and there is a relationship between **variables**

5. **Curve Sketching Using Differentiation**, where we begin to learn how to model the behaviour **variables**

More Curve Sketching Using Differentiation

16. **Applied Maximum and Minimum Problems**, which is a vital application of differentiation **Radius**
of Curvature, which shows how a curve is almost part of a circle in a local region

t.: Statement of the Problem

Differentiation is a technique which can be used for analyzing the way in which functions **change**. In particular, it measures how rapidly a function is changing at any point. This research **tends** to examine the differential calculus and its various applications in various fields, solving problems using differentiation. This work is to show the importance of differentiation, that it is **not** limited to mathematics alone, it is applied in our day to day life, it has its own share in our sciences motion, economic, chemistry. Etc.).

3 Purpose of the Study

The purpose of this project was to analyze operational principles of differentiation in calculus and problems faced by mathematicians and scientists

IA Specific Objectives

- i. To show that differentiation is not limited to mathematics alone.
- ii. To relate differentiation to velocity and acceleration in motion.
- iii. To relate differentiation in calculating rate of change of chemical reactions.

1.5 Research Questions

- i. How is differentiation applied in daily life activities?
- ii. How is differentiation related to velocity and acceleration in motion?
- iii. How can differentiation be related in calculating rate of change of chemical reactions?

1.6 Scope of the Study

This research work gave a vivid look at differentiation and its application.

Significance of the Study

study will act as a requirement for the partial fulfillment of award of bachelors in science

with Education

will help the entire community to understand how they can apply differentiation in

daily life

s Definition of Key Terms

Mathematics: is the branch of science concerned with number, quantity and space.

Calculus: This is the mathematics of change and motion. This implies that calculus is the type of mathematics that deals with rate of change.

Delta: Means change of variable, it could be written as Δ or δ

variable: A variable is a symbol such as x that may take any value in some specified set of

number

Function: A function is a set of ordered pairs of number (x, y) . To each value of the first variable x this corresponds to a unique value of the second variable y

Continuous function: A function of which is defined in some neighborhood of c is said to be continuous at C provided

$$\lim_{x \rightarrow c} f(x) = f(c)$$

explicit functions: If $Y = x^2 - 4x + 2$, Y is completely defined in terms of x and Y is called an **explicit** function.

Implicit function: This is a kind of equation with more than one variable that is having two variables known as ' y ' and ' x ' e.g. $Y = 4xy + x$ or $Y = 5x + 8$

Independent variable: The variable x which yields the first of the two numbers in the ordered pair (x, y) is often called independent variable or argument of the function f .

Dependent variable: The second variable Y is called the dependent variable.

CHAPTER TWO: LITERATURE REVIEW

2M Latroduction

-- us, historically known as infinitesimal calculus, is a mathematical discipline focused on --
:::ctions, derivations, integrals and infinite series.
--> eading up to the notion of function, derivatives and integral were developed throughout __ --=
~enrury but the decisive step was made by Isaac Newton and Gottfried Leibniz.

2I Techniques of differentiation.

~ - -::: **explores** various rules including the product, quotient, chain, and power, exponential
a: igarithmie rules.

u..I The product rule.

Iisis :J.Sed to find the derivative of the product of two functions U and V where U and V are
t=:f.-erent functions. If y is the product of two functions, $y = UV$

21.2 Quotient rule.

Tis is used to find derivatives of quotient of two functions U and V. that's if y is the quotient of

$$- . u \\ ::.mcnons, y =$$

: .. 1.3 Power rule

±ere. **we** find the derivative of the function raise to the nth power as the product of **n**, the function
raise to the power (**n-1**)

21.+ Sum rule.

iere we find the derivative of the sum of functions as the sum of the derivatives ofrespective **f tion**.

We saw that if y is the sum of different functions U, V, W, then

$$\frac{dy}{dx} = \frac{dU}{dx} + \frac{dV}{dx} + \frac{dW}{dx},$$

21.5 Chain rule

Exponential rule

was used in finding the derivatives of e raised to the power of a function, we use this rule to the derivatives of functions like $y = e^{f(x)}$

Lagrange's rule

is used to determine the derivative of natural logarithm of a function. We are able to find the derivative of functions $y = \ln f(x)$

~ Applications of differentiation

In Isaac Newton's day, one of the biggest problems was poor **navigation at sea**. Before **calculus** was developed, the stars were vital for navigation. Shipwrecks occurred because the ship was not where the captain thought it should be. There was not a good enough understanding of how the Earth, stars and planets moved with respect to each other. **Calculus** (differentiation and integration) was developed to improve this understanding. Differentiation and integration can help us solve many types of **real-world problems**. We use the **derivative** to determine the **maximum and minimum values** of particular functions (e.g. cost, strength, amount of material used in a building, profit, loss, etc.). Derivatives are met in many **engineering and science problems**, especially when modeling the behavior of moving objects.

23 Ancient Greek Precursors of the Calculus

Zeno mathematicians are credited with a significant use of infinitesimals.

Democritus is the first person recorded to consider seriously the division of objects into an **infinite** number of cross-sections, but his inability to rationalize discrete cross-section with a smooth slope prevented him from accepting the idea, at approximately the same time.

Zeno of Elea discredited infinitesimals further by his articulation of the paradoxes which they

create.

Archimedes and later Eudoxus are generally credited with implementing the method of exhaustion which made it possible to compute the area and volume of regions and solids by breaking them up into an infinite number of recognizable shapes.

Archimedes of Syracuse developed this method further, while also inventing heuristic method resemble modern day concept somewhat. It was not until the time of Newton that these methods were incorporated into a general framework of integral calculus.

It should not be thought that infinitesimals were put on a rigorous footing during this time,

by when it was supplemented by a proper geometric proof would Greek mathematicians accept proposition as true.

Pioneers of modern calculus

In the 17th century, European mathematicians Isaac barrow, Rene Descartes, Blaise Pascal, John Wallis and others discussed the idea of a derivative. In particular, Fermat developed a method for determining maxima, minima and tangents to various curves that was equivalent to differentiation.

Isaac Newton would later write that his own early ideas about calculus came directly from Fermat's way of drawing tangents

On the integral side Cavalieri developed his method of indivisibles in the 1630s and 40s, providing a modern form of the ancient Greek method of exhaustion and computing Cavalieri's quadrature formula, the area under the curves x^n of higher degree, which had previously only been computed for the parabola by Archimedes.

Torricelli extended this work to other curves such as cycloid and then the formula was generalized to fractional and negative powers by Wallis in 1656.

In 1659 treatise, Fermat was credited with an ingenious trick for evaluating the integral of any power function directly.

Fermat also obtained a technique for finding the centers of gravity of various plane and solid figures, which influenced further work in quadrature.

James Gregory influenced by Fermat's contributions both to tangency and to quadrature, was then able to prove a restricted version of the second fundamental theorem of calculus in the

17.

~ - - i-: $\frac{d}{dx}$. The first full proof of fundamental theorem of calculus was given by Isaac

Newton and Leibniz building on this work independently developed the surrounding of infinitesimal calculus in the late 17 century.

Leibniz did a great deal of work with developing consistent and useful notation and

Neon provided some of the most important applications to physics, especially of integral calculus.

3ore Newton and Leibniz the word "calculus" was a general term used to refer to anybody of mathematics, but in the following years, "calculus" became a popular term for a field of arhematics based upon their insight.

The work of both Newton and Leibniz is reflected in the notation used today.

Newton introduced the notation f' for the derivative of function f .

eibniz introduced the symbol for the integral and wrote the derivative of a function y of the clle x as both of which are still in use today

CHAPTER THREE

METHODOLOGY

3.1 Introduction

Chapter contains the methods and procedure used in differentiation and its application

3.1.1 Research Design

The study adopted a descriptive survey. Descriptive survey design is used in preliminary and

exploratory studies to allow the researcher gather information, summarize, present and **interpret** it for the purpose of clarification. It also allowed the researcher to describe record, **analyze** and report conditions that exist or existed. This design allowed the researcher to

both numerical and descriptive data that was used in measuring correlation variables. Descriptive survey **generate** research is intended to produce statistical information **Analysis** of differentiation and its **between** application. The field survey implied the process of **gaining** insight into the general picture of a situation, without utilizing the entire population.

3.2 Study Area

This study was carried out in Kabale municipality Kabale district; Kabale district is located in **south** western Uganda bordering Rwanda in the south Rukiga district in the East, Rubanda **district** in the north .

3.2 Population of the study

The study population consisted of workers in different Institutions in Kabale Municipality where **differentiation** can be applied. In this case the researcher considered learning in Kabale Municipality where she considered Kabale University as her case study.

3.2.1 Target population

The study population consisted of Kabale University community which included 490 students **and** 10 mathematics lecturers. Therefore the researcher considered 500 respondents

3.2.2 Sample size

The survey used a sample size determined using the Sloven's formula Altare et al. (2003).

$$n = \frac{N}{1 + N(e^2)}$$

= Sample Size 19.

= **cal** population (500)

= margin of error (5% or 0.05)

$$\frac{500}{1+500(0.05^2)}$$

$$1+1=1.25$$

---= ---,22222222222 ~222

-**zfore** the sample population was 222

:E... S:imple Selection

rp1erandom sampling and purposive sampling was used .

331Sample selection methods.

pleerandom sampling was used for students and purposive sampling was used to select m±hematics lecturers

5...: **n . .ua** collection methods and tools.

TEis **study** used questionnaires, documentation and interview guides to collect quantitative and **litative** data required for the study. Qualitative research consisted of detailed notation of **zeravior** events and contexts surrounding the event and behavior. I went through the questions

_: **die** respondent from both institutions to ensure a common understanding of the **estions** and ability to answer them correctly. They were open and closed ended

35.1Questionnaire method

The method was used to get information from the respondents where by open ended and closed .
"-clquestions were written on pieces of paper called questionnaires and filled by the c""""C:-Qndents. The method was used because it was less expensive.

3.5.2 Documentation

This was done by reviewing of books in the library and internet

3.5.3 Interviewing

Interviews were administered to the respondents, this is mainly to seek feelings and opinions from the respondents, I used interviewing because it always gives firsthand information. Open ended interviews were used.

3.6 Research procedure.

The researcher got a letter from Kabale University which was used to introduce her and enable her to get permission from the University Secretary. Pilot study was carried out in Kabale University community. Adjustments were made on the instruments where it was necessary.

3. 7 Ethical considerations.

In this case, the researcher treated people with respect; she also ensured that the procedures were reasonable and fairly administered. Full informed consent was obtained and privacy and confidentiality of the research participants was guided. I explained the real purpose and the use of the research to participants. Silenced voices were included to ensure that the groups marginalized in the society were considered and a mechanism was identified which published the research to enable the linking of research results to daily life. The information that was gathered from the subjects was confidential.

3.8 Data analysis

I employed content analysis method to analyze data; the emerging themes and opinions were properly analyzed and compared with the information from the literature review. Data was analyzed manually where frequencies, distribution tables and percentages were used to present the analyzed data from the field.

3.10. Limitations of the Study

Exploring the possible confounds and conducting a thorough investigation into the construct validity of information given by respondents was difficult and required an additional research. The research could perhaps assume a qualitative nature, and would probe people's concepts about personality resilience, comprehensibility, manageability and so on.

Interviews and panel discussions with people from the University community may helped but in spite of getting a research permit and letters of introduction from university Secretary, suspicion of the area of research also could cause unnecessary delays. Differentiation is not understood by most people

CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND INTERPRETATION OF FINDINGS

4.1 Introduction

This chapter covers the presentation and analysis of data that was collected using questionnaires. The variables covered were Differentiation and its application. 22 questionnaires were given to the respondents and all were returned. The chapter highlights the Respondent's Bio data. Discussion and analysis of the different responses to some key questions is also done in this chapter following the research objectives given below; to show that differentiation is not limited to mathematics alone, to relate differentiation to velocity and acceleration in motion, to relate differentiation in calculating rate of change of chemical reactions and How differentiation affects performance of demand and supply between buyers and sellers in economics

4.2 Back ground information of respondents.

This section shows the gender of the respondents, marital status, age bracket and highest level of education attained

4.2.1 Gender of respondents

Respondents were asked to state their gender and the following data was obtained.

Table 1: Gender of respondents

Valid	Gender	Frequency	Percentage	Cumulative Percentage
	Male	142	64	64
	Female	80	36	100
	Total	222	100	

Source: Primary Data 2019

From table 1, it can be noted that the council employs both the male and female though the majority were Male being represented by 64% while female are represented by 36%. This implies that both the Male and the female participated in the study although there was a difference of 28 %. It is an indication that Kabale University employment and admission are not biased by gender

4.3 Findings on application of differentiation

Table 2 : Application of differentiation

Motivation levels		Frequency	Percentage	Cumulative Percentage
	Agree	112	50.5	50.5
	Not sure	30	13.5	64
	Disagree	40	18.0	82
	Strongly disagree	40	18.0	100
	Total	222	100	

Source: primary data 2019

Table 5 shows that 50.5% strongly agree that differentiation can be applied in other fields, 13.5% not sure, 18.0% disagree that differentiation can be applied in other fields and only 18.0% strongly disagree that differentiation can be applied in other fields. This implies that the differentiation can be applied in other fields like trade, banking, education, transport and many other sectors

4.4 Findings on the relationship between differentiation, velocity and acceleration in motion.

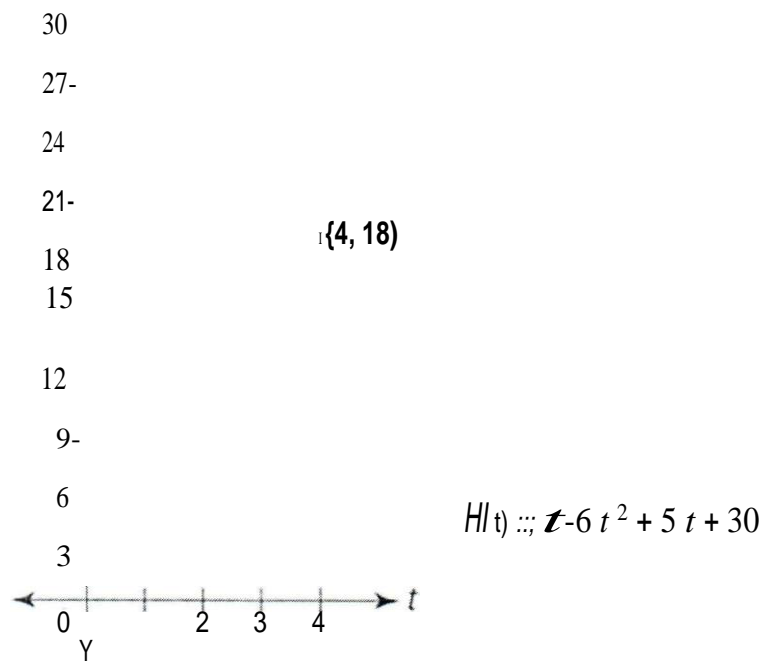
Every time you get in your car, you witness differentiation first hand. Your speed is the first derivative of your position. And when you step on the accelerator or the brake accelerating or decelerating you experience a second derivative.

If a function gives the position of something as a function of time, the first derivative gives its velocity, and the second derivative gives its acceleration. So, you differentiate position to get velocity, and you differentiate velocity to get acceleration.

Here's an example. A yo-yo moves straight up and down. Its height above the ground, as a function of time, is given by the function, Where t is in seconds and $H(t)$ is in inches. At $t = 0$, it's 30 inches above the ground, and after 4 seconds, it's at height of 18 inches.

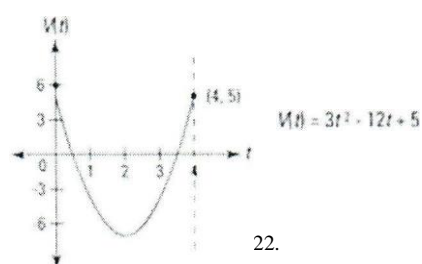
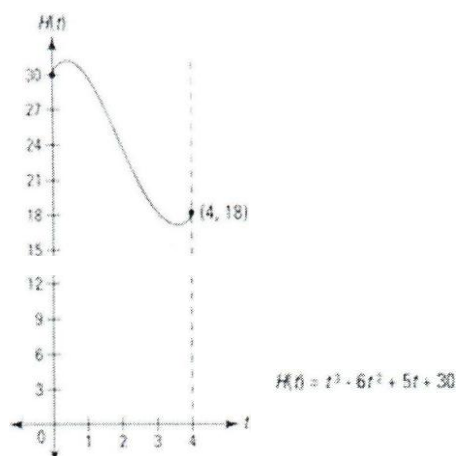
21.

H(t)



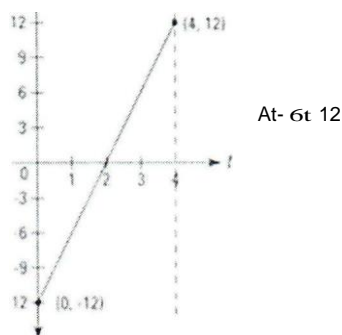
The yo-yo's height, from 0 to 4 seconds.

Velocity, $V(t)$ is the derivative of position (height, in this problem), and acceleration, $A(t)$, is the derivative of velocity. Thus



22.

Au



The graphs of the yo-yo's height, velocity, and acceleration functions from 0 to 4 seconds. **Velocity versus speed**

Your friends won't complain or even notice if you use the words "velocity" and "speed" interchangeably, but your friendly mathematician will complain. Here's the difference.

For the velocity function in the figure above, upward motion is defined as a positive velocity and downward velocity is defined as a negative velocity this is the standard way velocity is treated in most calculus and physics problems. (If the motion is horizontal, going right is a positive velocity and going left is a negative velocity.)

Speed, on the other hand, is always positive (or zero). If a car goes by at 50 mph, for instance, you say its speed is 50, and you mean positive 50, regardless of whether it's going to the right or the left. For velocity, the direction matters; for speed it doesn't. In everyday life, speed is a simpler idea than velocity because it agrees with common sense. But in calculus, speed is actually the trickier idea because it doesn't fit nicely in the three-function scheme shown in the figure above.

You've got to keep the velocity-speed distinction in mind when analyzing velocity and acceleration. For example, if an object is going down (or to the left) faster and faster, its speed is increasing, but its velocity is decreasing because its velocity is becoming a bigger negative (and bigger negatives are smaller numbers). This seems weird, but that's the way it works. And here's another strange thing: Acceleration is defined as the rate of change of velocity, not speed. So, if an object is slowing down while going in the downward direction, and thus has an increasing velocity because the velocity is becoming a smaller negative the object has a positive acceleration. In everyday English, you'd say the object is decelerating (slowing down), but in calculus class, you say that the object has a negative velocity and a positive acceleration.

Maximum and minimum height of $H(t)$ occur at the local extreme you see in the above figure. To locate them, set the derivative of $H(t)$ - that's $V(t)$ - equal to zero and solve.

These two numbers are the zeros of $V(t)$ and the t -coordinates that's time-coordinates of the max and min of $H(t)$, which you can see in the second figure. In other words, these are the times when the yo-yo reaches its maximum and minimum heights. Plug these numbers into $H(t)$ to obtain the heights:

$$H(0.47)=31.1$$

$$H(3.53)=16.9$$

So, the yo-yo gets as high as about 31.1 inches above the ground at $t = 0.47$ seconds and as low as about 16.9 inches at $t = 3.53$ seconds.

Total displacement is defined as the final position minus the initial position. So, because the yoyo starts at a height of 30 and ends at a height of 18,

$$\text{Total displacement} = 18 - 30 = -12.$$

This is negative because the net movement is downward.

Average velocity is given by total displacement divided by elapsed time. Thus,

$$\text{Average velocity} = \frac{12}{4} = -3$$

This negative answer tells you that the yo-yo is, on average, going down 3 inches per second. Maximum and minimum velocity of the yo-yo during the interval from 0 to 4 seconds are determined with the derivative of $V(t)$: Set the derivative of $V(t)$ equal to zero and solve:

$$\begin{aligned} V'(t) &= A(t) \\ 6t - 12 &= 0 \\ 6t &= 12 \\ t &= 2 \end{aligned}$$

Now, evaluate $V(t)$ at the critical number, 2, and at the interval's endpoints, 0 and 4:

$$\begin{aligned} V(0) &= 5 \\ V(2) &= -7 \\ V(4) &= 5 \end{aligned}$$

So, the yo-yo has a maximum velocity of 5 inches per second twice at both the beginning and the end of the interval. It reaches a minimum velocity of -7 inches per second at $t = 2$ seconds.

Total distance traveled is determined by adding up the distances traveled on each leg of the yoyo's trip: the up leg, the down leg, and the second up leg.

First, the yo-yo goes up from a height of 30 inches to about 31.1 inches (where the first turnaround point is). That's a distance of about 1.1 inches. Next, it goes down from about 31.1 to about 16.9 (the height of the second turn-around point). That's a distance of 31.1 minus 16.9, or about 14.2 inches. Finally, the yo-yo goes up again from about 16.9 inches to its final height of

18 inches. That's another 1.1 inches. Add these three distances to obtain the total distance traveled: $-1.1 + -14.2 + 1.1 = 16.4$ inches.

Average speed is given by the total distance traveled divided by the elapsed time. Thus,

$$\begin{aligned} \text{Average speed} &= \frac{16.4}{4} \\ &= 4.1 \text{ inches per second} \end{aligned}$$

Maximum and minimum speed.

You previously determined the yo-yo's maximum velocity (5 inches per second) and its minimum velocity (-7 inches per second). A velocity of -7 is a speed of 7, so that's the yo-yo's maximum speed. Its minimum speed of zero occurs at the two turnaround points.

For a continuous velocity function, the minimum speed is zero whenever the maximum and minimum velocities are of opposite signs or when one of them is zero. When the maximum and minimum velocities are both positive and both negative, then the minimum speed is the lesser of the absolute values of the maximum and minimum velocities. In all cases, the maximum speed is the greater of the absolute values of the maximum and minimum velocities. Is that a mouthful or what?

Maximum and minimum acceleration may seem pointless when you can just look at the graph of $A(t)$ and see that the minimum acceleration of -12 occurs at the far left when $t = 0$ and that the acceleration then goes up to its maximum of 12 at the far right when $t = 4$. But it's not inconceivable that you'll get one of those incredibly demanding calculus teachers who has the nerve to require that you actually do the math and show your work - so bite the bullet and do it.

To find the acceleration's min and max from $t = 0$ to $t = 4$, set the derivative of $A(t)$ equal to zero and solve:

This equation, of course, has no solutions, so there are no critical numbers and thus the absolute extreme must occur at the interval's endpoints, 0 and 4.

You arrive at the answers you already knew.

Note that when the acceleration is negative on the interval $[0, 2)$ that means that the velocity is decreasing. When the acceleration is positive on the interval $(2, 4]$ the velocity is increasing.

Speeding up and slowing down. Figuring out when the yo-yo is speeding up and slowing down is probably more interesting and descriptive of its motion than the info above. An object is speeding up (what we call "acceleration" in everyday speech) whenever the velocity and the calculus acceleration are both positive and both negative. And an object is slowing down (what we call "deceleration") when the velocity and the calculus acceleration are of opposite signs.

Look at all three graphs in the figure above again. From $t = 0$ to about $t = 0.47$ (when the velocity is zero), the velocity is positive and the acceleration is negative, so the yo-yo is slowing down (until it reaches its maximum height). When $t = 0$, the deceleration is greatest (12 inches per second per second; the graph shows an acceleration of negative 12, but here we're calling it a deceleration so the 12 is positive). From about $t = 0.47$ to $t = 2$, both velocity and acceleration are negative, so the yo-yo is slowing down again (until it bottoms out at the lowest height). Finally, from about $t = 3.53$ to $t = 4$, both velocity and acceleration are positive, so the yo-yo is speeding up again. The yo-yo reaches its greatest acceleration of 12 inches per second per second at $t = 4$ seconds.

Tying it all together. Note the following connections among the three graphs in the figure above. The negative section of the graph of $A(t)$ - from $t = 0$ to $t = 2$ - corresponds to a decreasing section of the graph of $V(t)$ and a concave down section of the graph $H(t)$. The positive interval of the graph of $A(t)$ - from $t = 2$ to $t = 4$ - corresponds to an increasing interval on the graph of $V(t)$ and a concave up interval on the graph $H(t)$. When $t = 2$ seconds, $A(t)$ has a zero, $V(t)$ has a local minimum, and $H(t)$ has an inflection point.

4.5 Relationship between differentiation and calculating rate of change of chemical reactions

The rate of a chemical reaction is the change in concentration over the change in time and is a metric of the "speed" at which a chemical reaction occurs and can be defined in terms of two observables:

$-\frac{\Delta[\text{Reactants}]}{\Delta t(1)}$ The Rate of Disappearance of Reactants =

Note this is negative because it measures the rate of disappearance of the reactants.

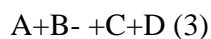
The Rate of Formation of Products $\frac{\Delta[\text{Products}]}{\Delta t(2)}$

This is the rate at which the products are formed.

They both are linked via the balanced chemical reactions and can both be used to measure the reaction rate.

Example

For example, in the simple reaction



The reaction rate can be defined thusly:

$$\text{Rate of disappearance of A} = -\frac{1}{M} \frac{d[A]}{dt}$$

$$\text{Rate of disappearance of B} = -\frac{1}{M} \frac{d[B]}{dt}$$

$$\text{Rate of formation of C} = \frac{1}{M} \frac{d[C]}{dt}$$

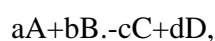
$$\text{Rate of formation of D} = \frac{1}{M} \frac{d[D]}{dt}$$

There are many factors that can either slow or speed up the rate of a chemical reaction such as temperature, pressure, concentration, and catalysts. The Rate of a Chemical Reaction is always positive. It can be confusing since the Rate of Disappearance is negative, however when you think about it, a rate should never be negative since the rate is describing how fast the concentration changes with time. The unit for the rate is Molarity per Seconds (M/s).

Reaction Rates from Non-unity Stoichiometric Coefficients

It does not matter whether an experimenter monitors the reagents or products. However, since reagents decrease during reaction, and products increase, there is a sign difference between the two rates. Reagent concentration decreases as the reaction proceeds, giving a negative number for the change in concentration. The products, on the other hand, increase concentration with time, giving a positive number. Since the convention is to express the rate of reaction as a positive number, to solve a problem, set the overall rate of the reaction equal to the negative of a reagent's disappearing rate.

The overall rate also depends on stoichiometric coefficients; consider the more general balanced equation



Where the lower case letters represent the coefficients of the balanced equation and the upper case letters (i.e. A) represent the molecular concentration. As with the example above, the rate of reaction can be defined with respect to loss of reactants or gain of products:

Rate of Disappearance of reactants:

$$\frac{-1a4[A]}{At} - \frac{1b4[B]}{At(8)}$$

Rate of Formation of product:

$$\frac{1c1[C]}{At} - \frac{1d1[D]}{At(9)}$$

Since Rate of Disappearance and Rate of Formation are equal

$$\frac{-1a1[A]}{At} = \frac{-1b1[B]}{At} = \frac{1c1[C]}{At} = \frac{1d1[D]}{At}$$

It is worth noting that the process of measuring the concentration can be greatly simplified by taking advantage of the different physical or chemical properties (i.e.: phase difference, reduction potential, etc.) of the reagents or products involved in the reaction by using the above methods. We have emphasized the importance of taking the sign of the reaction into account in order to get a positive reaction rate. Now, we will turn our attention to the importance of stoichiometric coefficients.

Even though the concentrations of A, B, C and D may all change at different rates, there is only one average rate of reaction. To get this unique rate, choose any one rate and divide it by the stoichiometric coefficient. When the reaction has the formula:

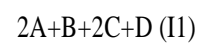


The general case of the unique average rate of reaction has the form:

$$\text{Rate of reaction} = \frac{-1a1[A]}{At} = \frac{-1b1[B]}{At} = \frac{1c1[C]}{At} = \frac{1d1[D]}{At}$$

Example

For the reaction:



Find the reaction rate given $[A] = 0.002, \text{ M}$ and $t = 77 \text{ s}$. **SOLUTION**

$$\text{Rate of reaction} = \frac{-\frac{1}{2} \frac{d[A]}{dt} = -\frac{1}{1} \frac{d[B]}{dt} = \frac{1}{2} \frac{d[C]}{dt} = \frac{1}{1} \frac{d[D]}{dt}}{12}$$

$$\text{Rate of disappearance of A} = \frac{-\frac{d[A]}{dt}}{12} = \frac{0.002 \text{ M}}{77 \text{ sec}} = -0.000026 \text{ M per sec}$$

$$\text{Rate of reaction} = \frac{1}{12} (\text{rate of disappearance of A}) = \frac{1}{12} (-0.000026 \text{ M per sec}) = -0.00000217 \text{ M per sec}$$

CHAPTER FIVE

SUMMARY OF FINDINGS, RECOMMENDATIONS AND CONCLUSIONS

5.1 Introduction

This chapter provides the summary of findings and recommendations from the study. The summary and recommendations are derived from the findings of the study which are presented in chapter four. Suggestions in areas thought necessary for further research also included.

5.2 Summary of major findings

5.2.1 Application of differentiation

Findings shows that showed most respondents 50% strongly agreed that differentiation can be applied in other fields, 13.5% not sure, 18.0% disagree that differentiation can be applied in other fields and only 18.0% strongly disagree that differentiation can be applied in other fields like trade, banking, education, transport and many other sectors 1

5.2.2 Relationship between Differentiation, velocity and acceleration in motion.

The relationship between differentiation and velocity and acceleration in motion were explained using the Example of YOYO .Maximum and minimum velocity of the yo-yo during the interval from 0 to 4 seconds are determined with the derivative of $V(t)$: Set the derivative of $V(t)$ that's $A(t)$ equal to zero and solve:

$$V'(t) = A(t)$$

$$A(t) = 6t - 12$$

$$6t - 12 = 0$$

$$6t = 12 \quad t = 2$$

Now, evaluate $V(t)$ at the critical number, 2, and at the interval's endpoints, 0 and 4:

$$V(0) = 5$$

$$V(2) = -7$$

$$F(4) = 5$$

So, the yo-yo has a maximum velocity of 5 inches per second twice at both the beginning and the end of the interval. It reaches a minimum velocity of -7 inches per second at $t = 2$ seconds.

Total distance traveled is determined by adding up the distances traveled on each leg of the yoyo's trip: the up leg, the down leg, and the second up leg.

5.2.3 Relationship between differentiation and calculating rate of change of chemical reactions

The rate of a chemical reaction is the change in concentration over the change in time and is a metric of the "speed" at which a chemical reaction occurs and can be defined in terms of two observables:

$$-\frac{1}{\Delta t} \frac{\Delta [\text{Reactants}]}{\Delta t} = \text{Rate of Disappearance of Reactants}$$

Note this is negative because it measures the rate of disappearance of the reactants.

$$\frac{1}{\Delta t} \frac{\Delta [\text{Products}]}{\Delta t}$$

This is the rate at which the products are formed.

They both are linked via the balanced chemical reactions and can both be used to measure the reaction rate.

5.3 Conclusion

Though mathematicians devote their time to study the concepts of rate of change, Rate of change gave birth to an aspect of calculus known as differentiation which is not applied in mathematics only but it is applied in real aspects of like Trade, Banking, Transport, Education among others therefore the researcher's major aim was to analyze Differentiation and its applications.

5.4 Recommendation

With regards to the findings and conclusions discussed in this study about the analysis of differentiation and its application, the following are the recommendations

Mathematicians should consider teaching Differentiation in a more practical way rather than theory.

The researcher also recommended that there should be sensitization of people about the significance of differentiation in real life, that differentiation is not applied to mathematics alone but also to other aspects of life.

5.5 Areas for Further Research

Trigonometric Identities for Pre-Calculus

How to Calculate Values for the Six Trigonometric Functions

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Newton began his work in 1666 and Leibniz began his in 1676. However, Leibniz published his first paper in 1684, predating Newton's publication in 1693. It is possible that Leibniz saw drafts of Newton's work in 1673 or 1676, or that Newton made use of Leibniz's work to refine his own. Both Newton and Leibniz claimed that the other plagiarized their respective works. This resulted in a bitter Newton Leibniz calculus controversy between the two men over who first invented calculus which shook the mathematical community in the early 18th century.

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