

Extending measurement range for three-dimensional structured light imaging with digital exponential fringe pattern

Kamagara Abel

Kabale University, Faculty of Engineering, Technology, Applied Fine Art and Design, Department of Electrical Engineering; Email: akamagara@kab.ac.ug

Abstract: In this paper, a method and an approach for intrinsically extending the measurement range for digital fringe projection profilometry with structured light imaging techniques is presented. This approach exploits the fact that at low levels of defocusing, exponential binary-coded fringe pattern exhibits a quasi-sinusoidal form having intact binary structures with reduced or negligible errors owing to high-order harmonic robustness during fringe generation. Experimental simulations and results show that within the desired region of defocus or at an extended measurement range, the proposed method exhibits a 45% comparative reduction in root-mean-square phase error hence improvement in final measurement result.

Keywords: Digital structured-light, 3D Measurement; Phase measurement profilometry

1. Introduction

Owing to recent advancement and leap in digital technology, optical three-dimensional profilometry based on digital sinusoidal fringe projection techniques has seen great advancement and application in both industry and scientific research and studies [1]. However, due to the inherent nonlinear gamma of most commercially available off-the-shelf digital projectors, it remains somewhat difficult to achieve high-quality three-dimensional profilometry using these techniques. In an endeavor to circumvent this problem, exponential fringe pattern projection [2-3] and squared binary defocusing [4-5] technique have been used and found to be robust to the nonlinear gamma problem. Similar to the method originally demonstrated by Su et al [6], squared binary defocusing strives to generate ideal pseudo sinusoidal fringe patterns by defocusing binary-structured ones. However, compared to conventional sinusoidal fringe patterns, squared binary defocusing has been borne with limitations: limited depth range; difficulty in quantifying the amount of defocus and high-frequency harmonic phase errors. These and many other limitations have limited the measurement quality of squared binary defocusing (SBD) technique. Nonetheless, effort has been made to circumvent some of these limitations and thence advance the technique. By combining intrinsic spectral sensitivities and a normed Fourier transform [7] of the defocused binary fringe pattern, the amount of defocus at each level of defocusing can be ascertained and subsequently an optimal degree of defocus required to generate sinusoidal fringe patterns.

In principle, the projector's optical engine is typically designed to generate large depth of focus, which is a great feature for conventional fringe projection methods where the projector is always nearly focused. In their nearly focused state [8], binary fringe patterns can actually produce depth range measurement similar to conventional sinusoidal fringe pattern generation techniques. Moreover, since most structured-light calibration methods tolerate minor deviations (similar to minimal low-pass filtering) from absolute focus, high accurate optical three-dimensional measurements can be achieved by applying these methods with a nearly focused projector. However, the binary defocusing method requires the projector to be out of focus which substantially causes measurement range to be compromised [5] given that image defocus limits the dynamic range of measurement.

With the recently developed [9-12] optimization methods on pattern generation to enhance the binary structured patterns such that low-pass filtering can neutralize harmonics and eliminate error-causing harmonics of binary fringe patterns at slight deviations from focus (resembling low levels of defocus or minimal low pass filtering), the depth range has been drastically increased. However, these methods exhibit various limitations ranging from limited fringe stripe width [9-10] slowed down or low speed measurement capabilities [11-12] hence sacrificing measuring speeds and unsuited for high speed imaging. This calls for adoption of a nearly focused squared binary with improved measurement accuracy without necessarily sacrificing measurement speeds for high-speed optical three-dimensional imaging.

In this paper, a new defocusing technique for three-dimensional profilometry with binary-coded exponential fringe patterns is proposed with an endeavor to reap and exploit the inherent merits of extended measurement depth range and low error-prone harmonics effects when in their nearly focused state i.e. with strong binary structures still somewhat evident. This technique is called binary-structured exponential defocusing (BED). The idea of enriching

measurement depth range with BED came from correlating a few observations: (1) too much image defocus tends to limit the dynamic measurement range; (2) seemingly sinusoidal exponential fringe pattern projection is less affected by high-order harmonics and (3) at low levels of defocus, exponential binary-coded fringe pattern seemingly exhibit a quasi-sinusoidal form with somewhat intact strong binary structures a vital trait for extended measurement range capabilities with binary defocusing technique. This technique has the leverage of enriching measurement depth range with projector using the binary defocusing technique. We believe, with this technique, an extended measurement depth range can be achieved with minimal modification in the optics of a projection system.

2. Principle and methodology

Three-dimensional profilometry with exponential fringe projection has been successfully demonstrated [2-3] to be robust to high-order harmonics and related phase errors. Theoretical analysis has demonstrated [8], that squared binary patterns in their nearly focused state have numerous frequency harmonics, and thus there is bound to be harmonic-related and residual phase errors in the final measurement result. Similar analogy has been extended to three-dimensional optical profilometry [3] with exponential fringe pattern and they have been found to be robust to high-order harmonics contrary to squared binary fringe patterns, hence there is no theoretical phase error in the final measurement result.

Binary patterns are used in squared binary defocusing to generate a phase and a code word. This means that only two gray-levels (0=dark and 255=bright) are used. With binary defocusing method, gamma calibration needn't be done. Generally, a normalized binary square wave $S(x)$ with a period of 2π can be expanded as a sum of odd harmonics in Fourier series as follows;

$$S(x) = 0.5 + \sum_{n=0}^{\infty} \frac{2}{(2n+1)\pi} \text{Sin}[(2n+1)x] \quad (1)$$

A low-pass digital filter is analogous to an optical defocusing system [13]. Since a projector lens is the same as low-pass filter, if squared binary wave is properly defocused, an ideal sinusoidal waveform can be generated [13]. The two-dimensional filtering or smoothing Gaussian function is given as;

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\pi\sigma^2}} \quad (2)$$

where σ is the standard deviation of zero-mean distribution. The intensity distribution $g(x)$ of a defocusing binary square wave can be described as a Fourier cosine series;

$$g(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \text{Cos}(2\pi k f_0 x) \rightarrow \forall k \in \text{odd} \quad (3)$$

where $g(x) = S(x) \otimes G(x, y)$.

f_0 is the fundamental spatial frequency, $a_k = \text{sinc}(k/2) (J_1(2\pi k \mu/2) / (\pi k \mu/2))$, J_1 is a Bessel function of the first kind and of the first order, μ is a defocusing parameter, $\mu = 2r/P$, P is the space period of the grating, and r is the radius of the circular exit pupil. As k increases, the component a_k drastically decreases or is dumped to a small value [13]. This emphatically indicates that a defocusing optical system i.e. is analogous to a low-pass filter attenuating or dumping higher-order harmonic components as previously stated. Similarly, a sinusoidal fringe pattern can be realized using an exponential function in the form of;

$$E(i, j)_{i=1:m; j=1:n} = \exp(bi) + \exp(2\pi * (f_1 * \frac{j}{n} + f_2 * \frac{i}{m})) \quad (4)$$

here, b is the fringe pattern intensity bias, m and n represent the pixel values in the generated fringe pattern while f_1 controls the vertical fringe pitch for vertical fringe pattern and f_2 controls the fringe pitch for horizontal fringe pattern.

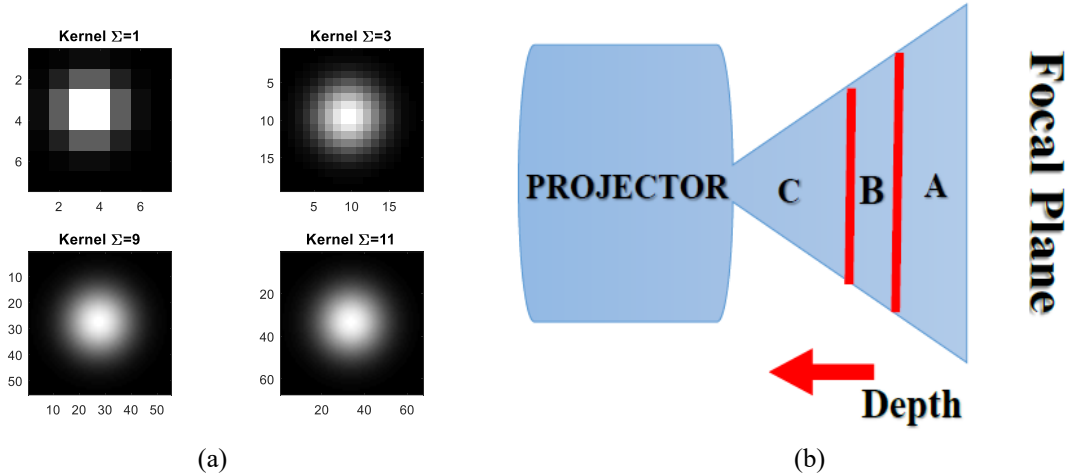


Fig.1 Schematic of model and behaviour of projector defocusing: (a) Gaussian blur kernels used to model projector defocusing; and (b) Behaviour of projector defocusing under increasing defocus depth.

The variables f_1 and f_2 can be modified depending on desired orientation of the fringe pattern. Using the expression of Eq. 4, the binary exponential fringe projection can be realized by coding the pattern with only two gray-levels (0=dark and 255=bright) and there after smooth or blur the coded fringe pattern with Gaussian function to realize binary exponential defocusing.

Analytically, image defocusing or blurring means that, instead of the light (photons or energy) from one pixel in the captured scene corresponding with the light at one pixel in the image, the energy from each pixel in the captured scene is spread out over multiple pixels in the image. The function that describes this spread is called the point spread function (PSF). The PSF is also the impulse response of the imaging system, and in applications dealing with blur, is sometimes referred to as the “blur kernel”. The size of the kernel controls the extent of blur or defocus. Figure 1 shows schematic of projector defocusing at different levels together with characteristic behaviour under different measurement range along the focal plane of the projector. In particular, Fig.1 (a) shows Gaussian blur kernels with varying sigma while Fig.1 (b) illustrates the operating behavior and principle of binary defocusing in relation to the measurement or depth range along the defocusing plane of the projector and used to model projector defocusing at varying measurement depth range. While region B develops high-quality sinusoidal fringe patterns, the patterns of region A retain binary structure with undesired levels of error, while signal-to-noise ratio of region C renders it unusable [5].

3.Experimental simulations

In Matlab environment using the image processing toolbox, we analyzed the effects of utilizing different levels of low defocus (i.e. nearly focused exponential binary patterns) synonymous with fringe patterns in region A along the projector plane on the quality of three-dimensional measurement using phase-shifting algorithms. Analytical comparison is done with conventional squared binary defocusing. Simulations and numerical results are presented to demonstrate its performance and comparative advantage to conventional squared binary defocusing. Figure 2 shows cross-sections of binary-structured exponential fringe pattern images when defocused at different levels by varying the size of the Gaussian blur kernel. Figure 3 shows the equivalent schematic using squared binary structured fringe pattern. A quasi three-dimensional measurement with a three-step least squares phase shifting algorithm (LS-PSA) is used to verify the performance of the proposed approach. We use the minimum 3-steps to acquire the fringe data as fast as possible especially for real-time profilometry. Moreover, LS-PSAs not only rejects more harmonics for a given number of phase-steps, but also have the highest signal-to-noise ratio for the quadrature-demodulated analytic signal [14]. This gives the proposed method a compounded robustness against high-order harmonics-related and residual errors [2-3]. Three phase-shifted fringe pattern images under different levels of defocus are projected onto a computer generated 3D object. Figures 4 (a)-(c) show the object used together with its reconstructed equivalent and phase profile of 128th row respectively by applying the spatial phase unwrapping algorithm to obtain and retrieve the continuous phase. Figures 4 (d)-(e) show the unwrapped phase profile of 128th row using defocus levels 1-2 with squared binary pattern while Fig. 4(f)-(g) show the equivalent phase profile using binary exponential fringe pattern within the same measurement range (region A) or defocusing depth range. Comparing Fig. 4(h) and Fig. 4(i) shows that when the projector is nearly in focus or at low degrees of defocus in desirable region A along the projector focal plane, the

proposed method surmounts the traditional squared binary defocusing technique.

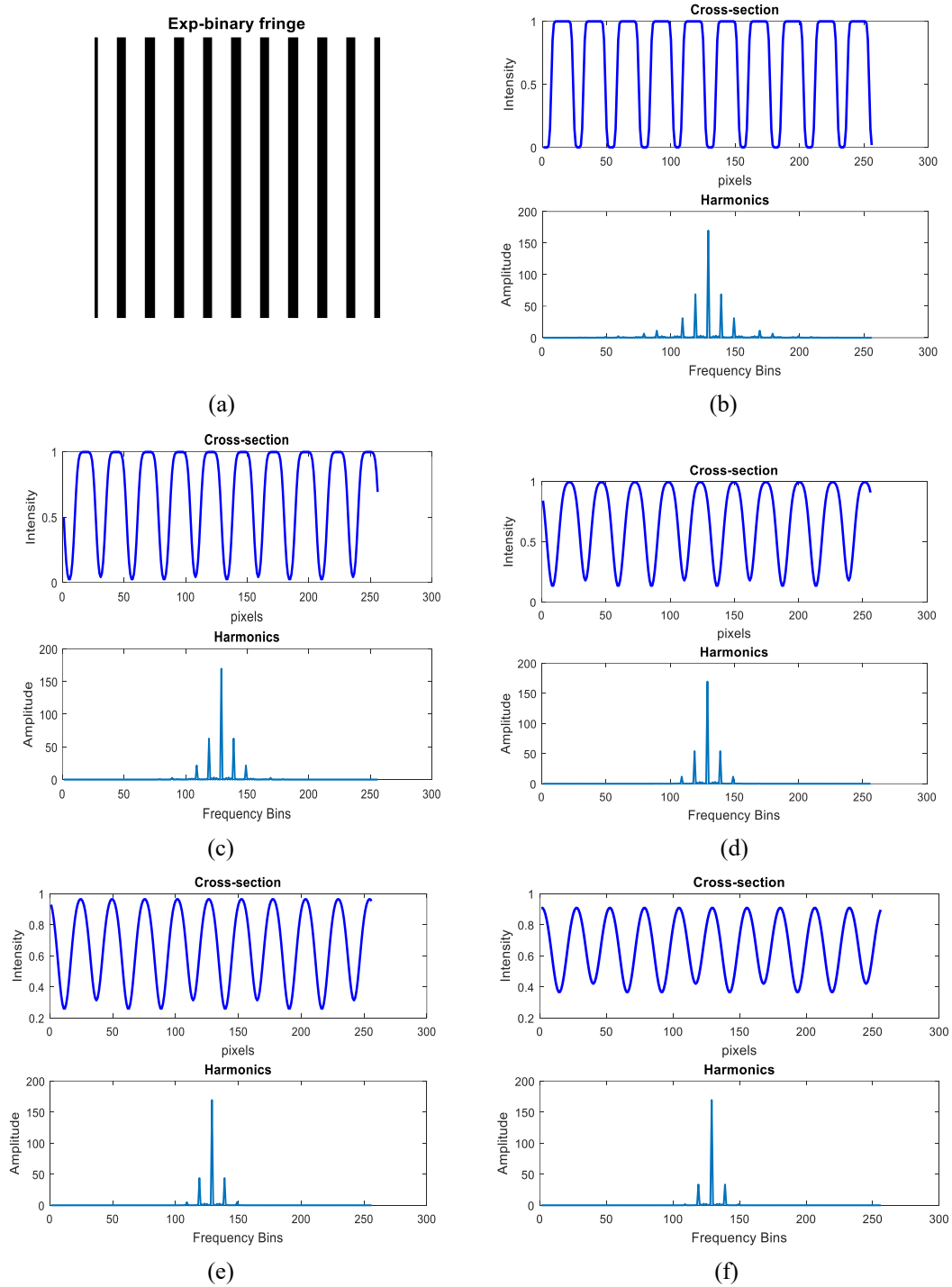


Fig. 2. Binary-structured Exponential fringe patterns at different defocusing levels, where level 1 is slightly defocused (region A) and level 5 is severely defocused (region C); (a) Binary-exponential fringe pattern; (b)-(f) corresponds to defocusing binary-structured exponential fringe pattern at defocus levels 1-5 (level 1 \ll level 5) respectively.

The proposed approach exhibits a 45% comparative reduction in the range $[-\pi, 0]$ or $[0, \pi]$ and a 20% comparative reduction in the range $[-\pi, \pi]$ in root-mean-square phase error hence improved accuracy in the final measurement result for three-dimensional shapes.

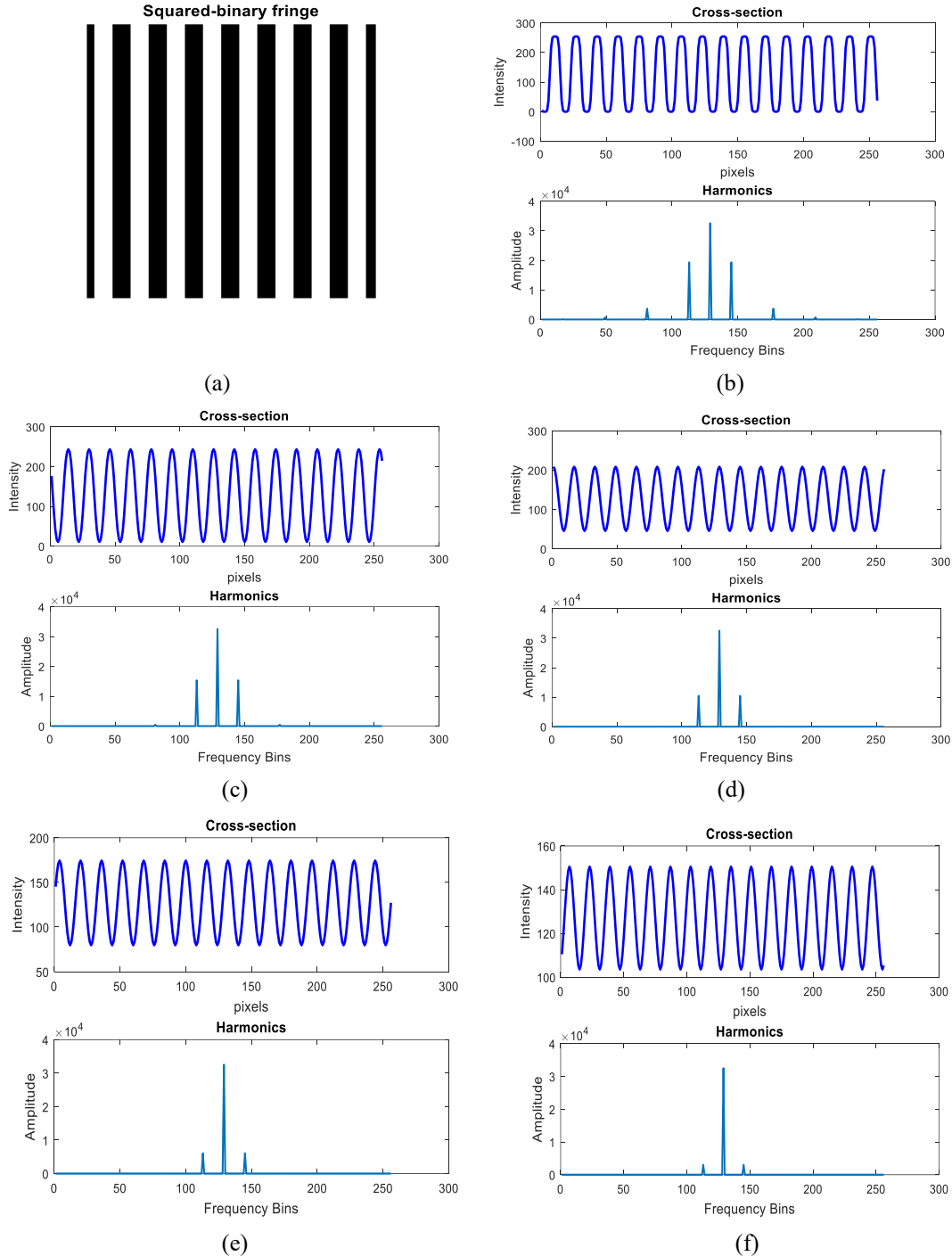
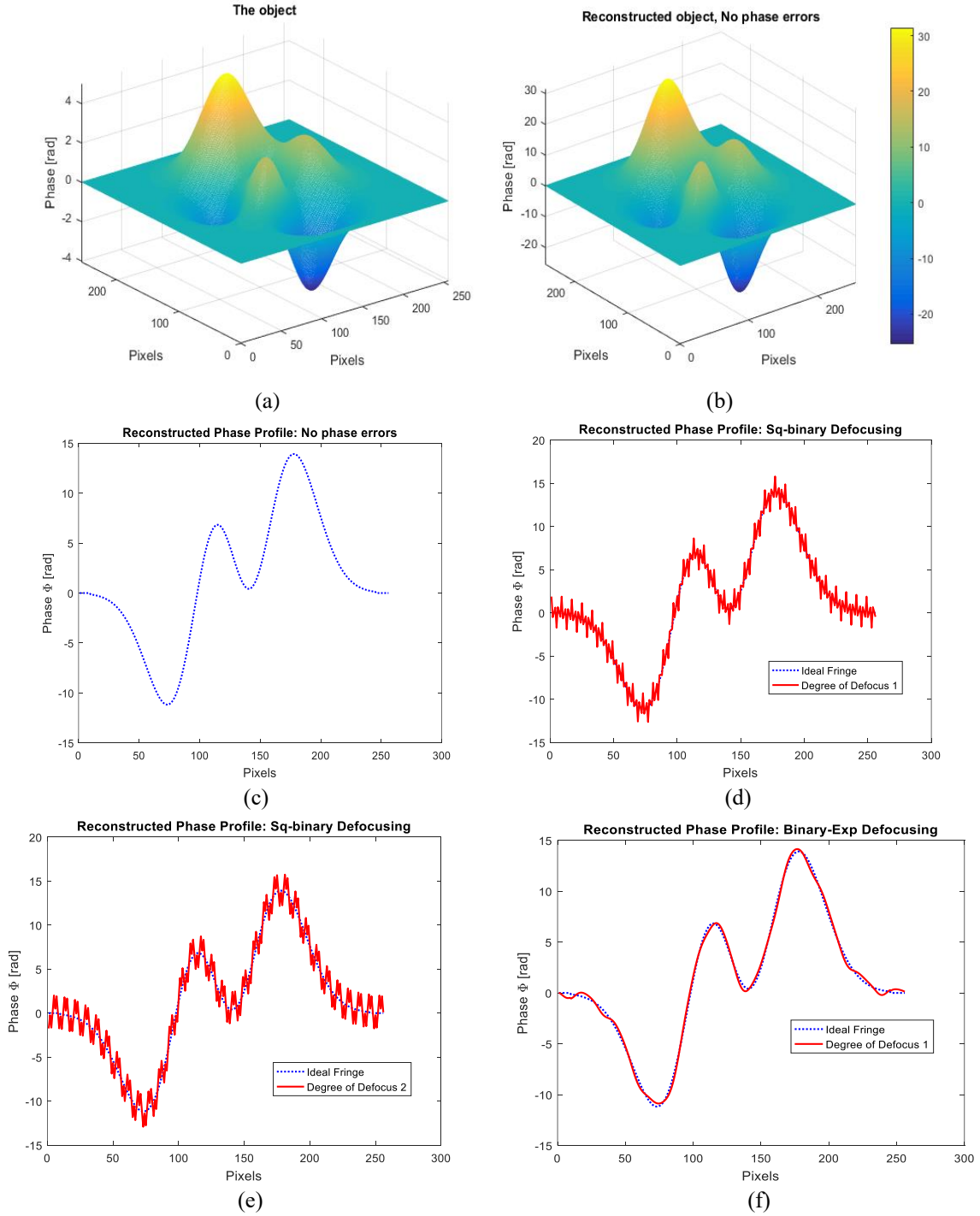


Fig. 3. Squared binary structured patterns at different defocusing levels, where level 1 is slightly defocused (region A) and level 5 is severely defocused (region C); (a) Square-binary fringe pattern; (b)-(f) corresponds to defocusing squared binary-structured fringe pattern at defocus levels 1-5 (level 1 \ll level 5) respectively.

To further analyze the performance and feasibility of proposed technique in contrast with the traditional squared binary defocusing method, 5 levels of defocusing are tested. Blur kernels of size 7 pixels i.e. defocus level 1 or defocus range in region A to 31 pixels i.e. defocus level 5 or defocus range in region C are used. Figure 4 (j) clearly depicts that the traditional squared binary defocusing exhibits more percentage phase error than the proposed approach at extended defocusing ranges. The major advantage of the proposed method is that it enables three-dimensional fringe

projection profilometry with LS-PSAs at extended fringe defocusing range void of non-negligible phase errors that plague the traditional squared binary defocusing method especially at larger defocusing range. It should be noted however that both methods produce same phase error under the same measurement conditions with optimal defocusing [7] applied.



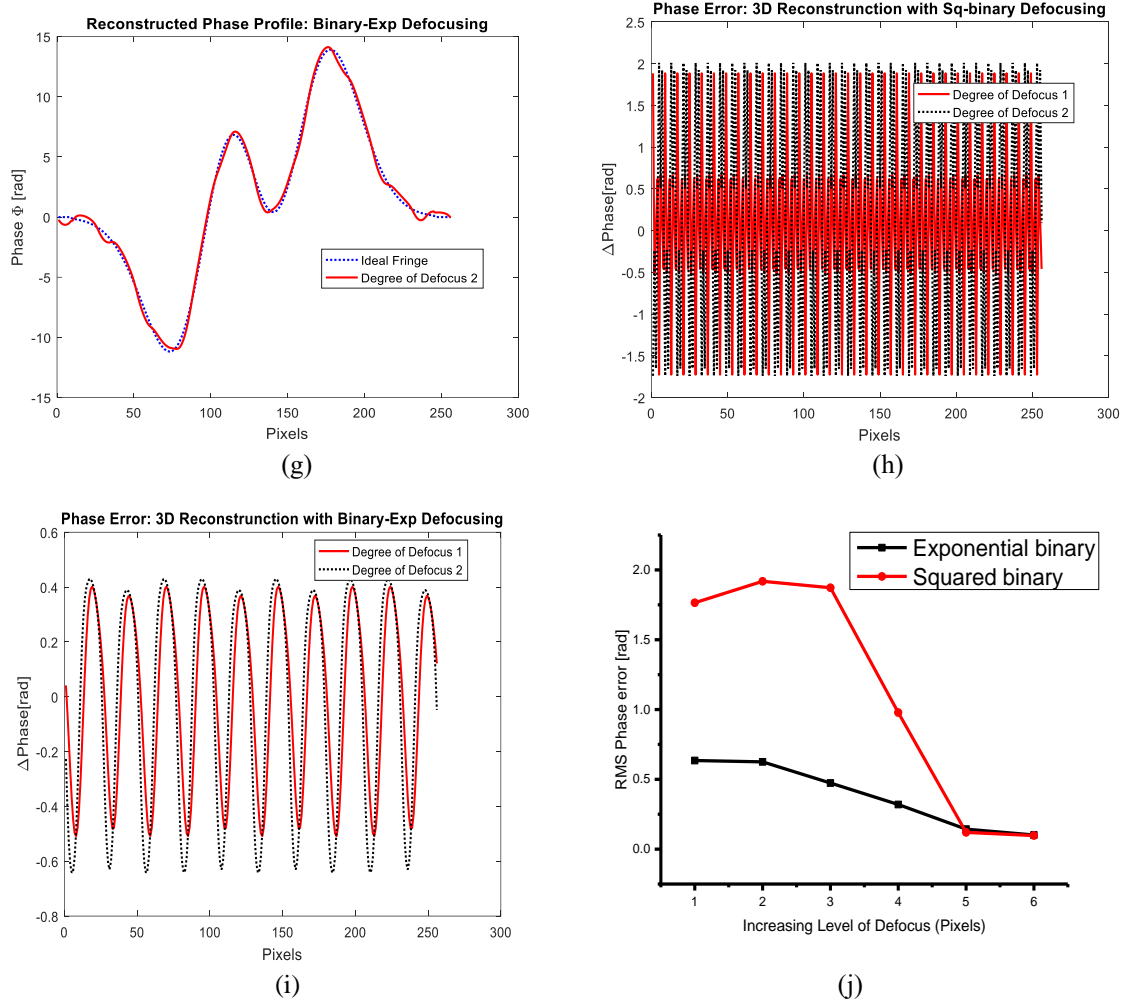


Fig. 4. Quasi three-dimensional profilometry with proposed and traditional squared binary defocusing: (a) the 3D object; (b) reconstructed object; (c) reconstructed phase profile of 128th row with ideal fringe; (d)-(e) reconstructed phase profile of 128th row with SBD; (f)-(g) reconstructed phase profile of 128th row with BED; (h) phase error of reconstructed object with SBD; (i) phase error of reconstructed object with BED; and (j) 128th row comparative phase error (RMSE) at different levels of defocusing. Exp. stands for exponential and Sq. stands for squared.

Moreover, at high levels of defocus (Fig. 1(b) in Region C), squared binary defocusing produces less phase errors though not a desirable feat since there is reduction in fringe image contrast and so the use of too much defocusing has two undesirable demerits of reduced defocusing depth range and lowered fringe pattern image contrast. This further underpins the inherent leverage of binary exponential defocusing over squared binary defocusing in terms of calibration difficulties and requirements.

4. Conclusion

A method and an approach for inherently generating quasi-sinusoidal fringe pattern at an extended measurement range using binary or projector defocusing for three-dimensional structured-light imaging with binary-structured exponential fringe pattern has been presented. As an illustration, numerical simulations of the root-mean-square phase error obtained at different levels of defocusing by applying varying pixel-sizes of low-pass Gaussian filter or blur kernel for both squared binary and binary-structured exponential fringe pattern images has been presented. Results have showed that at desirable fringe defocusing range or extended defocusing depth range i.e. when the projector is nearly in focus or at low degrees of defocus in desirable region along the focal plane of the projector, the proposed method i.e. binary-structured exponential defocusing (BED) outperforms squared binary defocusing technique (SBD) with a 45% comparative reduction in root-mean-square phase error (RMSE) as further exhibited in the final measurement results of the reconstructed three-dimensional phase profile. Therefore, compared to traditional squared binary

defocusing, the proposed method exhibits better percentage error metrics at extended measurement depth range along the defocusing plane hence an improved binary defocusing technique for digital fringe projection profilometry. Applying this method is promising for developing an inherently defocusing depth range enriched three-dimensional profilometry system without necessarily having to significantly modify the hard optics in the fringe projection system set-up.

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