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APPLICATION OF A DISCRETE-TIME SEMI-MARKOV MODEL TO THE STOCHASTIC FORECASTING OF CAPITAL ASSETS AS STOCK

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Abstract

In this paper, we developed and applied a stochastic model based on a discrete-time semi-Markov chain approach and its generalizations to study the high-frequency price dynamics of traded stocks. Semi-Markov is a stochastic process that generalizes both the Markov chain and the Markov renewal processes. It is well known that the performances of the stock market or factors that move stock prices are technical factors, fundamental factors, and market sentiments. The discrete-time semi-Markov model is applied on stock values of capital assets for both opening and closing prices for a specific period to predict the long-term behavior of the stock value price movement for the three states (bull market state, bear market state, and stagnant market state) for the process. The daily closing prices of stocks depend on the subsequent daily closing prices and there is a hope of recovering for capital asset stocks after the experience of unprecedented losses in stock values during the previous years. From the long run probabilities, the results showed that the probability of stock prices either increasing or decreasing is higher than being stable. So there is a high likelihood that stocks will not be stable in the long run. Thus, it is an indication that there is a high tendency for stock prices to fluctuate in the future than being stable.

1. Introduction

A stock market is an organized and regulated financial market where securities are traded at prices governed by the forces of demand and supply. Our main aim is to forecast the direction of future stock price patterns so that investors and traders can buy and sell stocks at lucrative prices. Thus, professional traders use fundamental and/or technical analysis to analyze and make investment decisions about stocks. Besides, in a bull market, since there are a low supply and high demand for securities, investors and traders have faith that the uptrend of stock prices will continue for over a long period and thus will be willing to sell a few and buy more securities. Conversely, in a bear market, there are low demand and high supply for securities which cause a negative sentiment in the market, leading to high chances for investors and traders to incur great losses since prices are continuously losing value. A stagnant market is when the stock market is neither increasing nor decreasing. When stock prices are initially in a bull state where it will remain there for a random amount of time having mean, then it will go to bear state where it will remain there for a random amount of time having a mean, then it will go to a stagnant state where it will remain for a random amount of time having a mean, then back to bull state again and so on. The process is a random variable if it is at a given state before making a move to a new state with mean and variance. Stock prices can move from a bull market state to at least one of the other states but cannot return to a bear market state thus leading to a transient period in the stock exchange. On the other hand, stock prices can move from a bull market state to at least one of the states and have at least one path to return to a bear market state causing a recurrent period in the stock exchange.

Semi-Markov processes are stochastic processes that generalize at the same time both Markov chains and renewal processes. In stochastic processes, Markov chains are crucial concepts to be considered. It satisfies the Markov property, meaning that knowing the current state of the system is sufficient to predict the future and so, the past and the future are not dependent on each other. Thus, no additional information or past data is needed to make predictions of the possible future after getting the current state of the process (Shehu et al. [8]). In forecasting the conditional probability of future events, the Markov chain being a stochastic process does not depend on past events or states but present states of the process. It is a mathematical system that experiences transitions from one state to another, according to some rules of probability. Mathematically, Markov chains are composed of state spaces, and these state spaces comprise vectors whose elements are all possible states of a stochastic variable due to the current state of the variable and transition matrix. The transition matrix incorporates the probabilities that variables will either remain or transit from one state to another. To compute probabilities of variables switching at the end of a particular state after n discrete partitions of time, there is a need to multiply the present state of the vector with transition matrix raised to power

n. There are quite different ways in terms of dealing with Markov chains and the renewal process depending on the nature of the parameters involved and areas of application. The methods are discrete-time Markov chains or continuous-time Markov chains. The former is when variable changes occur at a particular state and the latter is when variable changes are continuous. The changes of the system and probabilities associated with various state changes are characterized by a state space. A transition matrix relating the probabilities of particular transitions, and an initial state across the state space of the conditional probabilities for Markov chains are considered as transition probabilities. Markov chains can also have n-step transition probabilities besides one-step, in which it is the conditional probability that the process will be in state k after n-steps provided that it begins in state j at time s. The Markov renewal process can be a random process that generalizes the notion of Markov jump processes. The semi-Markov model attempts to generalize the Markov process by allowing a generally distributed holding time at each state. The model can help us know the result of how stock prices will move and be able to predict the future direction of stock price patterns of capital assets. One of the merits that the semi-Markov process has is that, for modeling the time to have a transition from one state to another, it allows the waiting time distributions. Markov models have elapsed on the waiting time distribution from one state to another and should be described by using a memory-less distribution (Norris and Norris [6]).

2. Literature Review

The stock market is still an integral part of any economy for developmental purposes. Variation in the stock market influences the health of any economy, personal and corporate financial lives. It is risky to invest in stock markets as stock prices are difficult to forecast if proper and rational decisions are not taken into consideration. Therefore, smart and reliable models are needed in predicting stock price movements in stock markets for proper decision-making and profitable opportunities. The problems for stock market volatility, seasonality, and time dependence, the rest of the market, and economies have been a struggle that induced researchers both in academia and industries. The semi-Markov process is applied in many fields of study and it is not something new in finance (Agbam and Udo [1]). (Ky and Tuyen [5]) did a study on the Markov-fuzzy combination model for stock market data prediction. The Markov model pinpoints the patterns of the data and predicts future states. According to the experimental results of their proposed model, it shows that other forecasting models did not perform better than their model. Odhiambo et al. [7] modeled the effect of COVID-19 pandemic on Kenyan Gross Domestic Product (GDP) contributors. In addition, by looking at the proportion of the contributors at a steady-state, they found the ultimate effect of the pandemic to the top five key sectors of the Kenyan economy that contributes massively to GDP growth. The outcome of their study should help global economies to have an understanding of varieties of economic stimulus planning packages to launch in the 'hard-hit' sectors by reducing potential economic recessions. Anthony and Othieno [2] applied the semi-Markov model for a portfolio of consumer loans and decided that they do not only respond to better credit risk modeling but go more than predicting for periods beyond the required regulatory minimum of three years. Furthermore, semi-Markov framework gives a more accurate prediction of default probability, the extent of exposure and hence facilitates adequate capital provision before the occurrence of the loss event. The main advantage of semi-Markov is that it allows the use of any type of waiting-time distribution function for modeling the time of switching systems from one state to another. D'Amico and Petroni [4] did a study for traded stock price returns by using a semi-Markov model. They employed the method of discrete-time homogeneous semi-Markov process and Markov chains for intraday and overnight returns, respectively. The use of volatility autocorrelation function, first passage time distribution, and non-parametric test of the hypothesis have been proposed to test the semi-Markov hypothesis. Thus, having shown through Monte Carlo simulation that the semi-Markov model produces better results than the simple Markov model, it was finally concluded that in modeling stock prices, the semi-Markov approach is better. D'Amico et al. [3] studied the high frequency dynamics of financial volumes for traded stocks using the semi-

Markov approach. The results of the data have been obtained from the Italian stock market from the first of January 2007, until the end of December 2010. They advanced the usage of weighted-indexed semi-Markov chain models, for modeling high-frequency financial volumes. The weighted-indexed semi-Markov chain model was used to describe the intraday logarithmic change of volumes by assumption. Moreover, based on their assumptions given above, they have shown that the model can reproduce stylized facts about intraday periodicity, the dependence of time series, and volume asymmetry. Shehu et al. [8] studied high-frequency capital asset price dynamics of traded stocks by using a semi-Markov model. The method of the Markov process to describe the state of systems and the state transitions was proposed. The method of negative exponential distribution has been used in modeling the continuous-time stochastic process to model the time of occurrence of events in the process. They developed the semi-Markov model by employing transitions, exponential distribution, and interval transition probabilities. The exponential holding time to describe the holding time in each state before making a transition to another state was used and chi-square was employed to test for independence of the closing prices. D'Amico and Petroni [4] discussed traded stock price changes by using a semi-Markov model with memory. They employed the method of discrete-time homogeneous semi-Markov model for intraday returns which depends on memory index. The memory index was introduced due to periods of high and low volatility in the market. The index semi-Markov model for price returns was proposed to overcome the problem of low autocorrelation. They have shown that the transition probability function of the semi-Markov process satisfies a renewal-type equation. Then, they concluded that the semi-Markov kernel is influenced by the past volatility, in which the past volatility, which has been used as a memory index produced well the behavior of the market returns. It has been shown that semi-Markov models have been developed in predicting stock price movements in stock markets in the form of bull, stagnant, or bear market. The trend of the stock market study is a global phenomenon and the mathematical formulations are not something new.

3. Methodology

3.1. Discrete-time semi-Markov chains mathematical preliminaries

Semi-Markov process is a generalization of the Markov process and renewal process. Thus, Markov process is a sequence of discrete random variables $(z_0, z_1, ..., z_s)$ such that the conditional distribution property of z_{s+1} given that the sequence is dependent only on the value of z_s but not on $z_0, z_1, ..., z_{s-1}$. Meaning that for any set of values $g, h, ..., k$ in the discrete state space, we have

$$p_{jk} = P\{Z_{s+1} = k \mid Z_0 = g, ..., Z_s = j\}$$

$$= P\{Z_{s+1} = k \mid Z_s = j\} \text{ for } j, k = 1, 2, 3. \tag{3.1}$$

The probability of transiting to a future state does not depend on the past state but the current state of the system. The entries p_{jk} for the matrix P of the process are the transition probability matrices. If the transition probability depends on time, it is non-homogeneous otherwise it is homogeneous. We are dealing more with transition probabilities that depend on time. Given that $Z(s_1), Z(s_2), ..., Z(s_n)$ exhibits a semi-Markov process for the random variables $s_1, s_2, ..., s_n$ for $s \in T$, consider the stochastic process, (K_s, T_s), $s = 0, 1, 2, ...$ as a homogeneous Markov chain with phase space $X \times (0, \infty)$, where $x = \{1, 2, 3, ..., d\}$ having an initial distribution

$$p_{jk} = P\{K_0 = j, X_0\} \text{ for } j \in X,$$

and for $j, k \in X$, $s, t \geq 0$ in the transition probabilities. By referring to the semi-Markov chain that is associated with the Markov renewal chain, we have

$$K(s) = Z_{N(s)}, \quad s \geq 0. \tag{3.2}$$

In the semi-Markov process, Z is representing the state space as the set of all possible values of j, k and p_{jk} is for the probability that the process in state j transits to state k. We will be describing the state transition probabilities by using a semi-Markov process. At a time, due to the nature of the stock exchange, it is in our interest to analyze how movements of stock prices happen in-between states and how long it takes for the stock price to remain within a state before making a move to another state (holding times). Thus, we let $p_{jk} =$ probability that the stock prices will transit to state k given that they are in state j. The probabilities of the transition satisfy that $\sum_k^3 p_{jk} = 1,\ p_{jk} \geq 0,$ where $j, k = 1, 2, 3.$

The holding time as a random variable having a positive value is governed by a holding time distribution function f_{jk} for a state transition from j to k as:

$$P(T_{jk} = r) = f_{jk}(r); \quad j, k = 1, 2, 3; \quad r = 1, 2, 3, \tag{3.3}$$

where j, k are the states and r is the time. We have to identify three holding time distributions in order to describe the semi-Markov process by adding a fixed value s to the state transition probabilities. Let f_{jk} be the cumulative probability distribution of T_{jk} in which T_{jk} is synonymous to each value of $k = 1, 2, 3$ such that

$$\overline{F}_{jk}(s) = P(T_{jk} \leq t) = \sum_{r=0}^{s} f_{jk}(r), \tag{3.4}$$

where $\overline{F}_{jk}$ is the complementary probability distribution of T_{jk}. We can let n to be the waiting time of the stock price in state j before taking a transition out of state j and m to be the probability distribution function of the waiting time n. Then

$$m(r) = P(m > r) = \sum_{k=1}^{3} p_{jk} f_{jk}(r). \tag{3.5}$$

The cumulative probability distribution for the waiting time is given as:

$$M_j(r) = P(n \le s) = \sum_{r=1}^{s} M_j(r)$$

$$= \sum_{k=1}^{3} p_{jk} F_{jk}(r) \tag{3.6}$$

and the complementary cumulative probability distribution for the waiting time is also given as:

$$M_j(s) = P(n > s) = \sum_{r-1=s}^{s} m_j(r)$$

$$= \sum_{k=1}^{3} p_{jk} \overline{F}_{jk}. \tag{3.7}$$

We can let $\Psi_{jk}(s)$ to be the probability that the stock price in state *j* will now transit to state *k* in some days *t*. Then,

$$\Psi_{jk}(s) = \eta_{jk} \overline{M}_j(s) + \sum_{l=1}^{3} p_{jl} \sum_{r=1}^{s} f_{jl}(r) \Psi_{lk}(s-r). \tag{3.8}$$

3.2. Transition matrix and steady-state probabilities model assumptions

(1) The direction and movement of capital assets as stock prices are random variables in between bull, bear, and stagnant markets that are time parameters indexed in a stochastic process.

(2) The probability that the stock price in a bear market will either transit to a bull or stagnant market is not independent of the current state of a bear market in a midst unit (within a day).

(3) Moving from state *j* to at least one of the state *k* but cannot return to state *j*, then it is transient.

(4) Moving from state *j* to at least one of the state *k* and have at least one path to return to state *j*, then it is recurrent.

(5) A state at which the stock price values rise from the previous state is the bull market state.

(6) A state at which the stock price value is stable from the previous state is the stagnant market state.

(7) A state at which the stock price value drops from the previous state is the bear market state.

The three states for the process to depict stock price movements of capital assets as a bull, bear and, stagnant market states are as follows:

State 1. Bull market state.

State 2. Bear market state.

State 3. Stagnant market state.

Below are the possible transitions between the states:

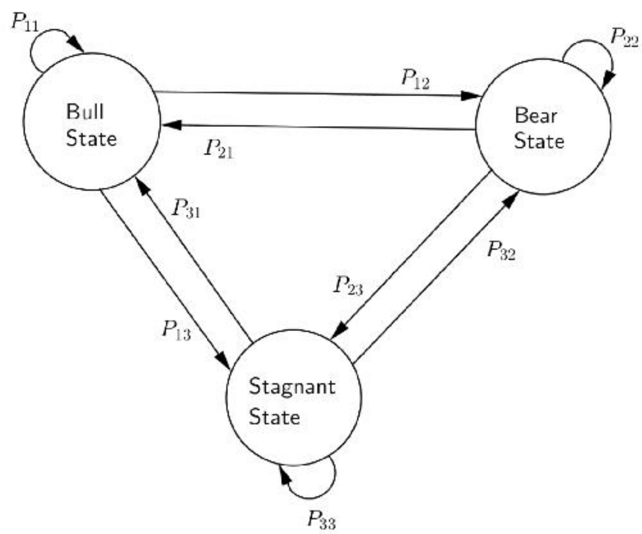

Figure 1. State transition diagram for stock price movement.

where

$p_{11} =$ probability of stock prices moving from bull market state to bull market state,

p_{33} = probability of stock prices moving from stagnant market state to stagnant market state,

p_{22} = probability of stock prices moving from bear market state to bear market state,

p_{13} = probability of stock prices moving from bull market state to stagnant market state,

p_{31} = probability of stock prices moving from stagnant market state to bull market state,

p_{32} = probability of stock prices moving from stagnant market state to bear market state,

p_{23} = probability of stock prices moving from bear market state to stagnant market state,

p_{12} = probability of stock prices moving from bull market state to bear market state,

p_{21} = probability of stock prices moving from bear market state to bull market state.

The matrix for the state transition probability of the process is given as follows:

$$p_{11} = \frac{n(11)}{n(11) + n(12) + n(13)}, \tag{3.9}$$

$$p_{12} = \frac{n(12)}{n(12) + n(11) + n(13)}, \tag{3.10}$$

$$p_{13} = 1 - [p_{11} + p_{12}], \tag{3.11}$$

$$p_{22} = \frac{n(22)}{n(21) + n(22) + n(23)}, \tag{3.12}$$

$$p_{21} = \frac{n(21)}{n(21) + n(22) + n(23)}, \tag{3.13}$$

$$p_{23} = 1 - [p_{21} + p_{22}], \tag{3.14}$$

$$p_{33} = \frac{n(33)}{n(31) + n(32) + n(33)}, \tag{3.15}$$

$$p_{31} = \frac{n(31)}{n(31) + n(32) + n(33)}, \tag{3.16}$$

$$p_{32} = 1 - [p_{31} + p_{33}], \tag{3.17}$$

where

$n(11) =$ number of times being in bull state at day s to bull state at day $s + 1$,

$n(12) =$ number of times being in bull state at day s to bear state at day $s + 1$,

$n(13) =$ number of times being in bull state at day s to stagnant state at day $s + 1$,

$n(21) =$ number of times being in bear state at day s to bull state at day $s + 1$,

$n(22) =$ number of times being in bear state at day s to bear state at day $s + 1$,

$n(23) =$ number of times being in bear state at day s to stagnant state at day $s + 1$,

$n(31) =$ number of times being in stagnant state at day s to bull state at day $s + 1$,

$n(32) =$ Number of times being in stagnant state at day s to bear state at day $s + 1$,

$n(33) =$ number of times being in stagnant state at day s to stagnant state at day $s + 1$.

The transition probability matrices are arranged in a matrix form as:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}.$$

The steady-state probability is: $[1_L, 2_L, 3_L]$, where

$1_L =$ The long run probability of the increasing stock price.

$2_L =$ The long run probability of the decreasing stock price.

$3_L =$ The long run probability of the stock price being stable.

We can compute the steady-state probability δ_k given the equations:

$$\sum_{k=1}^{3} \delta_k = 1,$$

$$\delta_k = \sum_{j}^{M} \delta_j p_{jk}, \quad \forall j,\ k = 1,\ 2,\ 3,$$

$$\delta_k \geq 0 \quad \forall k = 0, \ldots, M.$$

In matrix notation, we can have $\delta^T P = \delta^T$.

Now to solve for the steady-state probabilities, we can have $\delta^T P = \delta^T$ and $\sum_{j=1}^{3} \delta_j = 1$:

$$[\delta_1\ \delta_2\ \delta_3] \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = [\delta_1\ \delta_2\ \delta_3], \tag{3.18}$$

$$\delta_1 p_{11} + \delta_2 p_{21} + \delta_3 p_{31} = \delta_1, \tag{3.19}$$

$$\delta_1 p_{12} + \delta_2 p_{22} + \delta_3 p_{32} = \delta_2, \tag{3.20}$$

$$\delta_1 p_{13} + \delta_2 p_{23} + \delta_3 p_{33} = \delta_3. \tag{3.21}$$

By collecting like terms and factorizing, $\sum_{j=1}^{3} \delta_j = 0, \ \forall j = 1, 2, 3$ and $0 \le \delta_j \le 1,$ we have

$$\delta_1(p_{11} - 1) + \delta_2 p_{21} + \delta_3 p_{31} = 0, \tag{3.22}$$

$$\delta_1 p_{12} + \delta_2(p_{22} - 1) + \delta_3 p_{32} = 0, \tag{3.23}$$

$$\delta_1 p_{13} + \delta_2 p_{23} + \delta_3(p_{33} - 1) = 0. \tag{3.24}$$

The steady-state probability given the solution set of equation (3.22) to equation (3.24) is

$$\delta = (\delta_1, \delta_2, \delta_3).$$

Thus, the long-term probability will be considered as:

$$\lim_{n \to \infty} P^s, \tag{3.25}$$

where

P^s is the long-term transition probability from state j to k.

4. Results

4.1. Periodic trends for the stock price value

The stock prices for both opening and closing prices for the year 2019 are applied on the model. The stock market does not operate on public holidays and weekends. The closing prices are been grouped according to the market states as given in the table below.

Table 1. Transition count for the three states

Transition count	**Market states**
At least 100	Bull market state (state 1)
Between 98-99.99	Stagnant market state (state 3)
At most 97.99	Bear market state (state 2)

Table 1 describes the stock price movement as bull market state, bear market state, and stagnant market state. We can compute the probabilities of stock prices increasing, decreasing, and being stable, and we can forecast the future directions of stock price values using the equations for the transition probabilities.

Table 2 below shows stock price values for twelve months for the year 2019. The data is categorized into three states: bull, bear, and stagnant market states. The transition count for the stock prices can be estimated using relative frequency.

Table 2. Transition count for the year 2019 stock prices

State *j*/State *k*	**Bull state**	**Bear state**	**Stagnant state**	**Row total**
Bull state	94	3	0	97
Bear state	3	80	3	86
Stagnant state	0	3	64	67
Column total	97	86	67	250

The transition probability matrix is estimated using equation (3.9) to equation (3.17) and we have

$$P = \begin{bmatrix} 0.97 & 0.03 & 0 \\ 0.035 & 0.93 & 0.035 \\ 0 & 0.04 & 0.96 \end{bmatrix}.$$

The steady-state probabilities are computed given the solution set of equation (3.22) to equation (3.24) as

$$\delta = (0.3836,\ 0.3288,\ 0.2877),$$

where

$$\delta_1 = 0.3836,$$

$$\delta_2 = 0.3288,$$

$$\delta_3 = 0.2877.$$

The long-term probability matrix is estimated using equation (3.25) as

$$P^s = \begin{bmatrix} 0.3836 & 0.3288 & 0.2877 \\ 0.3836 & 0.3288 & 0.2877 \\ 0.3836 & 0.3288 & 0.2877 \end{bmatrix}.$$

4.2. χ^2 test of independence for closing stock prices

We want to test whether the closing prices are independent of each other using a chi-square distribution. The chi-square test of independence is

$$\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}, \tag{4.1}$$

where

O_i = Observed frequencies,

E_i = Expected frequencies.

Using equation (4.1), we can have

$$\chi_1^2 = \frac{(94 - 37.64)^2}{37.64} + \frac{(3 - 33.37)^2}{33.37} + \frac{(0 - 26.00)^2}{26.00} = 138.03,$$

$$\chi_2^2 = \frac{(3 - 33.37)^2}{33.37} + \frac{(80 - 29.58)^2}{29.58} + \frac{(3 - 23.05)^2}{23.05} = 131.02,$$

$$\chi_3^2 = \frac{(0 - 26.00)^2}{26.00} + \frac{(3 - 23.05)^2}{23.05} + \frac{(64 - 17.96)^2}{17.96} = 161.46.$$

Table 3. Test of independence for closing stock prices

States	χ^2	p-value
Bull state	138.03	0.001
Bear state	131.02	0.001
Stagnant state	161.46	0.001

The results in Table 3 showed that the daily closing prices for all the three states were not independent $(p < 0.05)$. It means that the daily closing price is dependent on the subsequent daily closing price. The transition probability matrix (P) and long-term probability matrix (P^s) as well as the steady-state probabilities give the probability of stocks being in the bull market state, bear market state, and stagnant market state. It showed that there is the hope for recovering Kenyan capital asset stocks that are listed at Nairobi Securities Exchange, after the experience of unprecedented loss in the stock value during the past years.

Thus, since the long run probability of the capital asset stocks for bull, bear, and stagnant are 0.3836, 0.3288, and 0.2877, respectively, there are, respectively, 38%, 33%, and 29% higher likelihood for stock prices to be in bull, bear, and stagnant market states in the long run. The probability of the stock to either increase (38%) or decrease (33%) is higher than being stable in the long run. This means, there will be more fluctuation at 5% in terms of stock price increment or decrement at Nairobi Securities Exchange and fewer chances for stock prices to be stable in the long run. This is a strong indication that stock prices will not be stable in the long run.

5. Conclusion

The data of stock prices for the year January 2019 to December 2019 used have been re-sampled into a 1-day frequency. Apart from public holidays and weekends, Nairobi Securities Exchange fixes opening prices of stocks at a random time after 9:30, then trading continues up to 2:30 pm. In the minute there are no transactions, the opening prices remain the same unless trading of securities occurs between buyers and sellers. The stock value of capital assets for both the opening and closing prices for the year 2019 revealed that Nairobi Securities Exchange does not operate on public holidays and weekends. The stocks of closing prices have been grouped into different categories as bull, bear, and stagnant states, and the holding time in each state before transiting into another state follows a Poisson process or distribution. The closing prices of the stocks for each of the states produce a p value that is less than the significant level of (0.05) and thus, closing prices were dependent. Meaning, the daily closing prices of the stock depend on the subsequent daily closing price and there is the hope of recovering for capital asset stocks after the experience of unprecedented losses in stock values in the previous years. From the long run probabilities, the results showed that the probability of stock prices either increasing or decreasing is higher than its being stable. Thus, it is an indication that there is a high tendency for stock prices to fluctuate than it being stable in the long run. According to the long run behavior of the capital asset stocks, there is a high likelihood that stock prices will not be stable in the future.

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