IST-Africa 2018 Conference Proceedings
Paul Cunningham and Miriam Cunningham (Eds)
IIMC International Information Management Corporation, 2018
ISBN: 978-1-905824-59-5

Stochastic Modeling of Internet Service for Profit Optimization in Uganda

Kizito Paul MUBIRU[1], Senfuka CHRISTOPHER[2], Maureen N. SSEMPIJJA[1]
[1]*Department of Mechanical & Production Engineering, Kyambogo University, Kyambogo, Uganda, Email: kizito.mubiru@yahoo.com*
[2]*Department of Engineering, Kabaale University, Kabaale, Uganda*

Abstract: We consider an internet cafe faced with an optimal choice of bandwidth for internet users under stochastic stationary demand. The choice is made over uniformly time horizons with a goal of optimizing profits. Considering customer demand, price and operating costs of internet service, we formulate a finite state Markov decision process model where states of a Markov chain represent possible states of demand for internet service. A profit matrix is generated; representing the long run measure of performance for the Markov decision process problem. The problem is to determine an optimal bandwidth adjustment policy so that the long run profits are maximized for a given state of demand. The bandwidth adjustment policies are determined using dynamic programming over a finite period planning horizon. Results from a case study demonstrate the existence of an optimal state-dependent option for bandwidth adjustment and profits in providing internet service.

Keywords: Bandwidth adjustment; internet service; modelling; profit optimization; stochastic demand

1. Introduction

The use of internet to access information is crucial for academic, business and social welfare in various communities all over the world. According to Doezi [1], the internet was one of the tools for bridging the gap between the ICT disadvantaged people. In today's e-business era, the internet allows virtually everything to be bought on-line and millions of people contact each other across the continents using internet service. As a vital means for job seekers, the internet serves as a platform where employer and employee can meet for possible recruitment. In the health sector, the internet allows healthy living to access medication and drugs using the e-pharmacy platform. Citizens from several countries have benefited in education where quality degrees can be earned from reputable institutions without one leaving his/her town or village [2]. In Uganda, use of the internet is picking at a steadily increasing rate. Uganda was one of the first countries in sub-Saharan Africa to gain full internet connectivity [3]. Major initiatives have been launched to bring internet services to rural areas in the country; partly funded by the highly successful operators through a universal service fund. A substantial number of cyber cafes are scattered within Kampala; Uganda's capital city. Stewart J [4] defines a cyber café as a café or shop open to the public; where a computer can be hired for specified periods to access the internet, write a CV or play a game. However, Jensen [5] notes that power outages pose a major problem against information/internet provision and use in African countries. Nyamoko, Richard and Makori [6] point out that internet café as a venture for business investment has essential areas to evaluate: the cybercafé system to invest in, how decisions made improve cybercafé business performance and the performance of cybercafé after implementation. However, Farbey [7] vividly shows how the use of such areas evaluated provide varying responses to different organizations. Situations vary from organization to organization and the range of circumstance the technique would be applicable is extremely wide. As a business investment venture, Berghout [8] concludes that both qualitative and quantitative methods are desirable.

2. Objectives of the Study

2.1 General Objective

The general objective of the study was to develop a mathematical model that optimize bandwidth adjustment decisions and profits of internet services under stochastic demand in Uganda.

2.2 Specific Objectives

Specifically, the study sought to attain the following objectives:

1. To select and define model variables and parameters
2. To determine demand transition and profit(reward) matrices of internet service
3. To develop a finite-period dynamic programming model that optimize bandwidth adjustment decisions and profits
4. To solve the model using a real life case study

3. Literature Review

Considering cost allocation and pricing of internet, Clark [9] gives the relationship between the range of service offered to users and the cost of providing these services. Based on his analysis, a new scheme for resource allocation and pricing is identified as well as the expected capacity allocation. This scheme is contrasted with a number of resource allocation schemes under consideration .Additional literature about internet resource pricing models, mechanisms and methods in Huan H,Xu K and Li Y[10] guide us to understand how to effectively use internet resources by examining pricing strategies, mechanisms and methods; and with the evolution of service types, several corresponding mechanisms which can ensure price implementation and resource allocation are considered with special reference to utility optimization economics. The bandwidth sourcing and task allocation problem in Turan, Nihatkasap and Hũseyin [11] provides a heuristic algorithm from firm's perspective at managerial level. The bandwidth provides selection and task allocation problem with stochastic constraints. Delay and jitter are considered random variables to capture stochastic nature of telecom network environment. Serafeimidis and Smithson [12] however points out the difficulty task to measure and identify the potential benefits and costs of an IT investment. It is also true that IT evaluation is complex and elusive and Dillon [13] argues that a phenomenal amount of money is lost because of inability of organizations to realize benefits.

While studies have tried to examine the dynamics of price and cost in internet service provision, a stochastic approach is sought to handle demand uncertainty among users of internet cafes with special reference to bandwidth provision as a profit maximization strategy.

4. Model Description

We consider an internet café that makes periodic adjustments of bandwidth to meet customer requirements for maximizing profits. The demand for internet service during each time period over a fixed planning horizon is described as either *favorable* (denoted by state

F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action namely; to adjust bandwidth for faster internet speed (a decision denoted by K=1) or not to adjust bandwidth (a decision denoted by K=0) during each time period over the planning horizon where K is a binary decision variable. Optimality is defined such that the maximum profits are accumulated at the end of N consecutive time periods spanning the planning horizon. In this paper, a two-period (N=2) planning period is considered.

4.1 Notation

Sets

i,j	Set of states of demand	K	Set of bandwidth adjustment policies

Parameters

Demand		Costs	
D	Demand matrix	C	Operational cost matrix
Q	Demand transition matrix	p_r	Price of internet service
Profits		Probabilities	
P	Profit matrix	Q^K_{ij}	Probability that demand changes from state i to state j given adjustment policy K
e	Expected profits		
a	Accumulated profits		

Others

R	Sales revenue	F	Favorable demand
n,N	Stages	U	Unfavorable demand
M	Customer matrix		

4.2 Finite-Period Dynamic Programming Model

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal bandwidth adjustment policy can be expressed as a finite period dynamic programming model. Assuming $g_n(i)$ denotes the optimal expected profits accumulated at the end of periods $n, n+1, \ldots\ldots\ldots\ldots N$ given that the state of the system at the beginning of period n is iε{F,U}.The recursive equation relating g_n and g_{n+1} is

$$g_n(i) = max_K[Q^K_{iF}P^K_{ij} + g_{n+1}(F), Q^K_{iU}P^K_{ij} + g_{n+1}(U)]$$

iε{F,U},Kε{0,1} , n=1,2,…….N (1)

together with the conditions

$$g_{N+1}(F) = g_{N+1}(U) = 0$$

This recursive relationship may be justified by noting that the cumulative profits P^K_{ij} + $g_{N+1}(j)$ resulting from state jε{F,U} at the beginning of period *n+1* from state i ε{F,U} at the beginning of period *n* occurs with probability Q^K_{ij}

Clearly, $e^K = [Q^K][P^K]^T$, Kε{0,1}
(2)

where "T" denotes matrix transposition.Hence, the dynamic programming recursive equations

$$g_N(i) = max_K[e_i^K + Q_{iF}^K g_{N+1}(F) + Q_{iU}^K g_{N+1}(U)] \quad (3)$$

$$g_N(i) = max_K[e_i^K] \quad (4)$$

result where (4) represents the markov chain stable state.

4.3 Computing Q^K and P^K

The demand transition probability from state i ε{F,U} to state j ε{F,U},given adjustment policy K may be taken as the number of customers observed with demand is initially in state i and later with demand changing to state j divided by the number of customers over all states.

That is

$$Q_{ij}^K = M_{ij}^K/[M_{iF}^K + M_{iU}^K] \qquad iε\{F,U\},Kε\{0,1\} \quad (5)$$

The profits can be expressed as the difference between sales revenue and operating costs

That is

$$P_{ij}^K = R_{ij}^K - C_{ij}^K \quad \text{where} \quad R_{ij}^K = p_r[D_{ij}^K] \quad (6)$$

5. Optimization

5.1 Optimization during Period 1

When demand is *favourable* (ie. in state F), the optimal bandwidth adjustment policy during period 1 is

$$K = \begin{cases} 1 & if \quad e_F^1 > e_F^0 \\ 0 & if \quad e_F^1 \le e_F^0 \end{cases}$$

The associated profits are then

$$g_1(F) = \begin{cases} e_F^1 & if \quad K = 1 \\ e_F^0 & if \quad K = 0 \end{cases}$$

Similarly, when demand is *unfavourable* (ie. in state U), the optimal bandwidth adjustment policy during period 1 is

$$K = \begin{cases} 1 & if \quad e_U^1 > e_U^0 \\ 0 & if \quad e_U^1 \le e_U^0 \end{cases}$$

In this case,the associated profits are

$$g_1(U) = \begin{cases} e_U^1 & if \quad K = 1 \\ e_U^0 & if \quad K = 0 \end{cases}$$

5.2 Optimization during Period 2

Using (3) and (4), and recalling that a^k_i denotes the already accumulated profits at the end of period 1, as a result of decisions made during that period, it follow that

$$a_F^K = e_i^K + Q_{iF}^K max[e_F^1, e_F^0] + Q_{iU}^K max[e_U^1, e_U^0]$$

$$a_F^K = e_i^K + Q_{iF}^K g_2(F) + Q_{iU}^K g_2(U)$$

Therefore when demand is *favourable* (ie. in state F), the optimal adjustment policy during period 2 is

$$K = \begin{cases} 1 & if \quad a_F^1 > a_F^0 \\ 0 & if \quad a_F^1 \le a_F^0 \end{cases}$$

while the optimal profits are

$$g_2(F) = \begin{cases} a_F^1 & if \quad K = 1 \\ a_F^0 & if \quad K = 0 \end{cases}$$

Similarly, when demand is *unfavourable* (ie. in state (U), the optimal adjustment policy during period 2 is

$$K = \begin{cases} 1 & if \quad a_U^1 > a_U^0 \\ 0 & if \quad a_U^1 \le a_U^0 \end{cases}$$

In this case, the associated profits are

$$g_2(U) = \begin{cases} a_U^1 & if \quad K = 1 \\ a_U^0 & if \quad K = 0 \end{cases}$$

6. A Case Study about Zion Internet Cafe in Uganda

In order to demonstrate use of the model in §3-4, a case study from *Zion Internet Cafe* in Uganda is presented in this section. Customers come to Zion Internet Cafe and the demand for internet service fluctuates every week based on the bandwidth and speed realised at the internet cafe. The manager's goal is to maximize profits when demand is favourable (state F) or unfavourable (state U) and hence, seek decision support in terms of an optimal bandwidth adjustment policy and the associated profits for offering internet service in a two-week planning horizon.

6.1 Data Collection

Samples of customers, demand (in minutes) and operational costs of internet service (in US$) were collected. The state transitions of demand and the respective bandwidth adjustment policies were examined over twelve weeks. The data is presented in Tables 1-3.

Table 1: Customers versus state-transitions at Zion Internet Cafe

	Adjustment Policy 1		Adjustment Policy 0	
State of demand	F	U	F	U
F	20	5	18	3
U	15	8	12	6

Table 2: Demand (in minutes) versus state-transitions at Zion Internet Cafe

	Adjustment Policy 1		Adjustment Policy 0	
State of demand	F	U	F	U
F	4800	600	3000	1000
U	650	1200	950	850

Table 3: Operational costs (in US$) versus state-transitions at Zion Internet Cafe

	Adjustment Policy 1		Adjustment Policy 0	
State of demand	F	U	F	U
F	6	5	2.8	5.3
U	4.5	4.3	6.8	4

For any chosen bandwidth adjustment policy, the price of internet service (p_r) = US$0.0084 per minute

6.2 Computation of Model Parameters

Using (5) and (6), the state-transition matrices and profit matrices (in US$) were as follows

$$Q^1 = \begin{bmatrix} 0.800 & 0.200 \\ 0.652 & 0.348 \end{bmatrix} \qquad P^1 = \begin{bmatrix} 14.16 & 0.04 \\ 0.96 & 1.58 \end{bmatrix}$$

for the case when the bandwidth was adjusted (K=1) during week 1 while these matrices are given by

$$Q^0 = \begin{bmatrix} 0.857 & 0.143 \\ 0.667 & 0.333 \end{bmatrix} \qquad P^0 = \begin{bmatrix} 22.4 & 3.1 \\ 1.18 & 3.14 \end{bmatrix}$$

for the case when bandwidth was *not* adjusted(K=0) during week 1. When the bandwidth was adjusted, the matrices Q^1 and P^1 yield the expected profits (in US$) in week 1

$$e_F^1 = (0.800)(34.32) + (0.200)(0.04) = 27.464$$

$$e_U^1 = (0.652)(0.960) + (0.348)(1.58) = 1.176$$

When the bandwidth was *not* adjusted (K=0), the matrices Q^0 and P^0 yield the expected profits (in US$) in week1

$$e_F^0 = (0.857)(22.4) + (0.143)(3.1) = 19.640$$

$$e_U^0 = (0.667)(1.18) + (0.333)(3.14) = 1.833$$

When the bandwidth was adjusted (K=1), the accumulated profits (in US$) at the end of week 2 are calculated as follows

$$a_F^1 = 27.464 + (27.464)(0.800) + (1.833)(0.200) = 49.802$$

$$a_U^1 = 1.176 + (27.464)(0.652) + (1.833)(0.348) = 19.720$$

When the bandwidth was *not* adjusted (K=0), the accumulated profits (in US$) at the end of week 2 are

$$a_F^0 = 19.640 + (27.464)(0.857) + (1.833)(0.143) = 47.366$$

$$a_U^0 = 1.833 + (27.464)(0.667) + (1.833)(0.333) = 20.762$$

6.3 The Optimal Bandwidth Adjustment Policy

Week 1

Since 27.464>19.640, it follows that K=1 is an optimal bandwidth adjustment policy for week 1 with associated profits of 27.464 US$ for the case of favourable demand. Since 1.833>1.176, it follows that K=0 is an optimal bandwidth adjustment policy for week 1 with associated profits of 1.833 US$ for the case when demand is unfavourable.

Week 2

Since 49.802 >47.366, it follows that K=1 is an optimal bandwidth adjustment policy for week 2 with associated accumulated profits of 49.802US$ for the case of favourable demand. Since 20.762>19.720, it follows that K=0 is an optimal bandwidth adjustment policy for week 2 with associated accumulated profits of 20.762US$ for the case of unfavourable demand.

7. Conclusions and Discussion

A Markov decision model for choosing an optimal bandwidth adjustment policy in providing internet service under stochastic stationary demand was presented in this paper. The decision of whether on not to adjust the bandwidth for faster internet service and optimal profits is made using dynamic programming over a finite period planning horizon. Results from the model indicate optimal adjustment policies and profits at Zion Internet Cafe. As a profit maximization strategy for internet service delivery at internet cafes, computational efforts of using Markov decision process approach provide promising results. However, further extensions of the research are vital to analyze a considerable number of cybercafés as well as the impact of non stationary demand on the choice of bandwidth adjustment policies. In the same spirit, the model developed raises a number of salient issues to consider: power disruptions in offering internet service and customer response to abrupt changes in price and speed of internet service. Special interest is also sought in further extending the model by considering adjustment policies for optimal profits using continuous time markov chains. However, the model developed is to allow its application in the specific form in order to assist internet cafe managers in achieving optimal profits. As noted in the study, profit comparisons were vital in determining the optimal bandwidth adjustment policy. By the same token, classification of demand as a two-state Markov chain facilitated the modeling and optimization process of bandwidth adjustment policies for the specific case study considered.

The paper is therefore an eye opener to Africa-EU partnership in ICT development. European financing instruments can increase resources to foster support through investment grants, technical assistance, risk capital and other risk sharing instruments for ICT development in Africa. This can strengthen ICT sustainability in order to support achievement of Sustainable Development Goals(SDG) including breakthroughs in digital education-governance and healthcare. Despite the technical and financial challenges, internet and bandwidth have to grow and expand in Uganda and Africa as a whole. The need for cheaper access to high bandwidth of internet service is highly sought. Although the available bandwidth is limited and insufficient to meet existing demand in Uganda, novel solutions are needed to both per-customer bandwidth management and per-flow bandwidth management in internet cafes. This strategy will support sustainable ICT growth in African countries in order to enhance scientific and technical development of African economies.

Acknowledgements

The authors would like to thank the staff of Zion Internet Cafe whose data contributed to the stage of testing the developed model.

References

[1] Doezi Y (2000). "The Good Research Guide for small-scale Research projects "Open University Press, Buckingham

[2] Kuboye B.,Alese B.,Imasuen F.(2012) A twin Approach to Internet service provision in sparse rural community in Nigeria,*International Journal of Networks and Communications*,2(5),132-137.

[3] Review,Internet ,ICT,Broadband and consumer e-commrce in Uganda,June 2017.

[4] Stewart J & Williams R (1998),"The co-evaluation of society and multimedia Technology", 16(3)

[5] Jensen M (2002) The African Internet,Astatus report http//www.3.sn.apc.org/africa/afstat.htm

[6]Nyamoko O,Richard R & Makoi A (2014) User Activity-Based Decision Support for cost optimization of cybercafe investment,*International Journal of Scientific and Technology Research,*3(5),15-17.

[7]Farbey B,Land F & Target D.(2002) IT Investment:Astudy of Methods and Practice,Butterworth and Heinemann

[8]Bergout E (1997) Evaluation of Information System Proposal.Design of a Decision Support Method.Doctoral Dissertation.Delft University of Technology

[9] Clark D (1995) Model for Cost Allocation and pricing in the internet,*The Journal of Electronic Publishing*,1(1-2);presented at MIT workshop on Internet Economics.

[10] Huan H,Ke X & Ying L(2011),Internet Resource Pricing Models,Mechanisms and Methods

[11] Hűseyin H,Nihatkasap,Hűseyinsavaran A(2014). A Bandwidth sourcing and task allocation model in telecommunications under stochastic QoS guarentees.*Applied Soft Computing*, vol.24, 1112-1123.

[12] Sarafeimidis V & Smithson J (2008) Interpretive Information Systems Evaluation in Practice:Experience from case study.Computer science Report Series.Department of Computing,University of Surrey,CS-98-01.

[13] Dhillon G (2012) Interpreting key issues in IS/IT.Benefits Management.Proceedings of the 33rd Hawaii International Conference on System Science, Jan 04-07.