

Modeling Campaign Optimization Strategies in Political Elections under Uncertainty

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ABSTRACT

In most political campaigns, the overall goal of every candidate is to maximize the number of voters during the election exercise. In such an effort, cost effective methods in choosing the optimal campaign strategy are paramount. In this paper, a mathematical model is proposed that optimizes campaign strategies of a political candidate. Considering uncertainty in voter support and cost implications in holding political rallies, we formulate a finite state Markov decision process model where states of a Markov chain represent possible states of support among voters. Using daily equal intervals, the candidates' decision of whether or not to campaign and hold a political rally at a given location were made using discrete time Markov chains and dynamic programming over a finite period planning horizon. Empirical data was collected from two locations on a daily basis during the campaign exercise. The data collected was analyzed and tested to establish the optimal campaign strategy and costs at the respective locations. Results from the study indicated the existence of an optimal state-dependent campaign strategy and costs at the respective political rally locations.

Keywords : Campaign, Elections, Modeling, Optimization, Uncertainty

I. INTRODUCTION

In today's fast-paced and competitive political ground, the success of a political campaign demands cost-effective distribution of resources where political campaign stake place among prospective voters. When the voting population is deeply divided, costs of running political campaigns increase enormously for aspiring candidates. Although different populations located in different environments can be tailored with different campaigning strategies, the optimality of each strategy is trivial for best results. In real world campaign contexts, some candidates postpone campaigns as a cost-minimization strategy; which may be risky if the opposing candidates are financially

secure. However, the goal of political campaigns is to maximize the probability of victory at least-cost. In most political campaigns, changes are realized on how people vote after changing voter attitudes and perceptions of the running candidate.

In this paper, a novel stochastic model is proposed whose goal is to optimize campaign strategies of a political candidate as a cost-minimization strategy. The paper is organized as follows. After reviewing the relevant literature, a mathematical model is described where consideration is given to the process of estimating model parameters. The model is solved and applied to a special case study. Some final remarks finally follow. The major contribution of the

proposed model is to show how markov decision processes can be used to optimize campaign strategies of political candidates at various locations. More specifically,

1. We illustrate how the voter support matrix and campaign cost (reward) matrix can be computed
2. We show the computational procedure of expected and accumulated campaign costs
3. As a cost-minimization strategy, we determine the optimal least-cost campaign strategy at the designated locations

II. RELATED WORK

The effects of negative and positive attitudes on candidates (Malloy, Merkwowitz 2016) suggest why candidates continue to attack their opponents by considering real world campaign contexts which candidates work in competition with each other. Candidates have to react to the decisions of the opposing campaigns. Results suggest that it is never efficacious for candidates to run attack ads. Running positive ads can increase a candidate's margin of victory. Peterson (2014) illustrates the degree of uncertainty in campaigns and how such degree can change people's votes; although how campaigns have this effect is less well understood. The prevailing view is that these effects occur by changing the context of voter's attitudes and by changing the weights votes applied by these determinants of vote choice. The use of operations research in planning political campaign strategies (Barkan, Bruno 1972) indicated how costs of running campaigns have increased tremendously over the years and methods have been sought to make campaigns more efficient in their utilization of resources. In similar contexts, mathematical models for economic and political advertising campaigns were studied. To that note, Shane (1977) proposed a Game theoretic saddle point solution for the following four problems:

- i) How large should the total advertising budget be to maximize profits?

- ii) How should the budget be distributed in a differentiated market and how it is saved by this distribution?
- iii) How should one distribute advertising dollars in order to maximize one's expected total number of votes in a political campaign?
- iv) How should one allocate expenditures over time in order to maximize one's expected number of voters at election day?

The prevailing model, when tested, was found to yield a very high correlation between actual and predicted behavior. Belenky (2005) took a closer look at competitive strategies of US presidential candidates in election campaigns and showed that most problems can be formulated as discrete mathematical programming ones or as those with mixed variables; and indeed some problems can be formulated as game theory types. The campaign optimization problem was also handled through behavioral modeling and mobile network (Altshuler, Shmueli, Zykind et al 2006) where authors examined the use of available resources with the ultimate goal of winning. A mathematical model was proposed to compute an optimized campaign by automatically determining the number of interacting units, their type and how they should be allocated to different geographical regions in order to maximize the campaign's performance. The problem of predicting the winning candidate in Samuelson (2006) becomes complicated and this is illustrated by the 13 keys' model. This model outperformed its creator in which only two prominent forecasters got it right. The model used 13 'Yes-No' variables that reflect satisfaction with incumbent party but the poles were wrong. The probabilistic aspects in political campaigns and elections using the Bayesian prediction model as in (Rigdon, Jacobson, Cho, Sewell 2009) showed the closeness of previous presidential elections and the wide accessibility of data how it should change and how presidential election forecasting should be conducted. A Bayesian forecasting model was proposed that concentrated on the electoral college outcome and considered finer details such as third-

the beginning of period n is $i \in \{F, U\}$. The recursive equation relating g_n and g_{n+1} is

$$g_n(i, L) = \min_K [V_{iF}^K(L)C_{iF}^K(L) + g_{n+1}(F, L), V_{iU}^K(L)C_{iU}^K(L) + g_{n+1}(U, L)] \quad (1)$$

$i \in \{F, U\}$, $L = \{1, 2\}$, $K \in \{0, 1\}$ $n = 1, 2, \dots, \dots, N$

together with the conditions

$$g_{N+1}(F, L) = g_{N+1}(U, L) = 0$$

This recursive relationship may be justified by noting that the cumulative campaign costs $C_{ij}^K(L) + g_{N+1}(i, L)$ resulting from reaching state $j \in \{F, U\}$ at the start of period $n+1$ from state $i \in \{F, U\}$ at the start of period n occurs with probability $V_{ij}^K(L)$

$$e^k(L) = [V^k(L)]^T [C^k(L)] \quad L = \{1, 2\} \quad K \in \{0, 1\} \quad (2)$$

where “T” denotes matrix transposition. Hence, the dynamic programming recursive equations

$$g_{N+1}(i, L) = \min_K [e_i^K(L) + V_{iF}^K(L)g_{N+1}(F, L) + V_{iU}^K(L)g_{N+1}(U, L)] \quad (4)$$

$$g_N(i, L) = \min_K [e_i^K(L)]$$

result where (4) represents the Markov chain stable state.

3.4 Computing $V^K(L)$

The voter support transition probability from state $i \in \{F, U\}$ to state $j \in \{F, U\}$, given campaign strategy $K \in \{0, 1\}$ may be taken as the number of supporters observed at location L with support initially in state i and later with support changing to state j , divided by the sum of supporters over all states. That is,

$$V_{ij}^K(L) = S_{ij}^K(L) / [S_{iF}^K(L) + S_{iU}^K(L)] \quad (5)$$

$$i, j \in \{F, U\} \quad , \quad K \in \{0, 1\} \quad , \quad L = \{1, 2\}$$

IV. OPTIMIZATION

The optimal campaign strategy and costs are found in this section at location L during each period separately.

4.1 Optimization during period 1

When voter support is favorable (ie. In state F), the optimal campaign strategy and costs are

$$K = \begin{cases} 1 & \text{if } e_F^1(L) < e_F^0(L) \\ 0 & \text{if } e_F^1(L) \geq e_F^0(L) \end{cases}$$

and

$$g_1(F, L) = \begin{cases} e_F^1(L) & \text{if } K = 1 \\ e_F^0(L) & \text{if } K = 0 \end{cases}$$

respectively.

Similarly, when voter support is unfavorable (ie. In state U), the optimal campaigning strategy and costs during period 1 are

$$K = \begin{cases} 1 & \text{if } e_U^1(L) < e_U^0(L) \\ 0 & \text{if } e_U^1(L) \geq e_U^0(L) \end{cases}$$

and

$$g_1(U, L) = \begin{cases} e_U^1(L) & \text{if } K = 1 \\ e_U^0(L) & \text{if } K = 0 \end{cases}$$

respectively.

4.2 Optimization during period 2

Using (3) and (4) and recalling that $a^k_i(L)$ denotes the already accumulated campaign costs at location L during the end of period 1 it follows that

$$a_i^K(L) = e_{iF}^K(L) + V_{iF}^K(L) \min[e_{iF}^1(L), e_{iF}^0(L)] + V_{iU}^K(L) \min[e_{iU}^1(L), e_{iU}^0(L)]$$

Therefore, when voter support is favorable (ie. in state F), the optimal campaign strategy and costs during period 2 are

$$K = \begin{cases} 1 & \text{if } a_F^1(L) < a_F^0(L) \\ 0 & \text{if } a_F^1(L) \geq a_F^0(L) \end{cases}$$

and

$$g_2(F, L) = \begin{cases} a_F^1(L) & \text{if } K = 1 \\ a_F^0(L) & \text{if } K = 0 \end{cases}$$

respectively

Similarly, when voter support is unfavorable (ie. in state U), the optimal campaign strategy and costs during period 2 are

$$K = \begin{cases} 1 & \text{if } a_U^1(L) < a_U^0(L) \\ 0 & \text{if } a_U^1(L) \geq a_U^0(L) \end{cases}$$

and

$$g_2(U, L) = \begin{cases} a_U^1(L) & \text{if } K = 1 \\ a_U^0(L) & \text{if } K = 0 \end{cases}$$

respectively.

V. A CASE STUDY ABOUT POLITICAL CAMPAIGN STRATEGIES FOR LOCAL COUNCIL (LC) ELECTIONS IN UGANDA

the associated campaign costs in a two-day planning period.

In order to demonstrate use of the model in §3-4, a real case application for political candidature at two locations in Uganda are presented in this section. Support for the candidate fluctuates everyday among voters at both locations. The campaign team wants to minimize costs when support is either favorable (state F) or unfavorable (state U) and hence, seek decision support in terms of an optimal campaign strategy and

5.1 Data collection

Samples of supporters, and costs were taken as a result of state-transitions in voter support. These samples were taken for five weeks under the respective campaign strategies. The data is presented in Table 1 and Table 2.

Table 1. Supporters versus state transitions at Campaign locations

Campaign Location (L)	States (F/U)	Campaign strategy 1		Campaign strategy 0	
		F	U	F	U
1	F	100	40	75	60
	U	55	10	68	35
2	F	78	35	65	45
	U	45	20	80	30

Table 2. Costs(in US\$) versus state transitions at Campaign locations

Campaign Location (L)	States (F/U)	Campaign strategy 1		Campaign strategy 0	
		F	U	F	U
1	F	300	250	175	140
	U	100	90	200	110
2	F	150	200	80	130
	U	180	160	100	50

From Table 1, the supporter matrices are directly obtained for each respective location.

Location 1

$$S^1(1) = \begin{bmatrix} 100 & 40 \\ 35 & 10 \end{bmatrix} S^0(1) = \begin{bmatrix} 75 & 60 \\ 68 & 35 \end{bmatrix}$$

Location 2

$$S^1(2) = \begin{bmatrix} 78 & 35 \\ 45 & 20 \end{bmatrix} S^0(2) = \begin{bmatrix} 65 & 45 \\ 80 & 30 \end{bmatrix}$$

The campaign cost matrices are similarly obtained for each location using the data in Table 2.

Location 1

$$C^1(1) = \begin{bmatrix} 300 & 250 \\ 100 & 90 \end{bmatrix} C^0(1) = \begin{bmatrix} 175 & 140 \\ 200 & 110 \end{bmatrix}$$

Location 2

$$C^1(2) = \begin{bmatrix} 150 & 200 \\ 180 & 160 \end{bmatrix} C^0(2) = \begin{bmatrix} 80 & 130 \\ 100 & 50 \end{bmatrix}$$

5.2 Computation of Model Parameters

Using (5),the voter support transition matrices at each respective location are

$$V^1(1) = \begin{bmatrix} 0.7143 & 0.2857 \\ 0.8462 & 0.1538 \end{bmatrix} V^1(2) = \begin{bmatrix} 0.6903 & 0.3097 \\ 0.6923 & 0.3077 \end{bmatrix}$$

for the case of holding a political campaign(K=1) while these matrices are given by

$$V^0(1) = \begin{bmatrix} 0.5556 & 0.4444 \\ 0.6602 & 0.3398 \end{bmatrix} V^0(2) = \begin{bmatrix} 0.5909 & 0.4091 \\ 0.7273 & 0.2727 \end{bmatrix}$$

for the case of *not* holding a political campaign(K=0)

When a political campaign was held(K=1),the voter support matrices and campaign cost matrices yield the expected costs(in US\$) during day 1 at the two locations:

Location 1

$$e_F^1(1) = (0.7143)(300) + (0.2857)(250) = 286.72$$

$$e_U^1(1) = (0.8462)(100) + (0.1538)(80) = 96.92$$

Location 2

$$e_F^1(2) = (0.6903)(150) + (0.3097)(200) = 165.49$$

$$e_U^1(2) = (0.6923)(180) + (0.3077)(160) = 173.85$$

When a political campaigns was *not* held(K=0),the voter support matrices and campaign cost matrices yield the expected costs(in US\$) during day 1 at the two locations

Location 1

$$e_F^0(1) = (0.5556)(175) + (0.4444)(140) = 159.45$$

$$e_U^0(1) = (0.6602)(200) + (0.3398)(110) = 169.42$$

Location 2

$$e_F^0(2) = (0.5909)(80) + (0.4091)(130) = 100.46$$

$$e_U^0(2) = (0.7273)(100) + (0.2727)(50) = 86.37$$

For the case of holding a political campaign(K=1),the accumulated campaign costs(in US\$) at the end of day 2 are calculated for the two locations.

Location 1

$$a_F^1(1) = 286.72 + (0.7143)(159.45) + (0.2857)(96.92) = 428.31$$

$$a_U^1(1) = 96.92 + (0.8462)(159.45) + (0.1538)(96.92) = 246.75$$

Location 2

$$a_F^1(2) = 165.49 + (0.6903)(100.46) + (0.3097)(86.37) = 261.59$$

$$a_U^1(2) = 173.85 + (0.6923)(100.46) + (0.3077)(86.37) = 269.97$$

Similarly,for the case of *not* holding a political camapign(K=0),the accumulated campaign costs(in US\$) at the end of day 2 are calculated for the two locations.

Location 1

$$a_F^0(1) = 159.45 + (0.5556)(159.45) + (0.4444)(92.92) = 291.11$$

$$a_U^0(1) = 169.42 + (0.6602)(159.45) + (0.3198)(92.92) = 307.62$$

Location 2

$$a_F^0(2) = 100.46 + (0.5909)(100.46) + (0.4091)(86.37) = 195.16$$

$$a_U^0(2) = 86.37 + (0.7273)(100.46) + (0.2727)(86.37) = 182.98$$

5.3 The Optimal Campaign Strategy

Location 1(Day 1)

At location 1, since $159.45 < 256.73$, it follows that $K=0$ is an optimal political campaign strategy for day 1 with associated campaign costs of 159.45 US\$ for the case of favorable voter support. Since $96.92 < 169.42$, it follows that $K=1$ is an optimal campaign strategy for day 1 with associated campaign costs of 96.92 US\$ for the case of unfavorable voter support.

Location 2(Day 1)

At location 2, since $100.46 < 165.44$, it follows that $K=0$ is an optimal campaign strategy for day 1 with associated campaign costs of 100.46 US\$ for the case of favorable voter support. Since $86.37 < 173.85$, it follows that $K=0$ is an optimal campaign strategy for day 1 with associated campaign costs of 86.37 US\$ for the case of unfavorable voter support.

Location 1(Day 2)

At location 1, since $291.11 < 428.31$, it follows that $K=0$ is an optimal campaign strategy for day 2 with associated accumulated campaign costs of 291.11 US\$ for the case of favorable voter support. Since $246.75 < 307.62$, it follows that $K=1$ is an optimal campaign strategy for day 2 with associated accumulated campaign costs of 246.75 US\$ for the case of unfavorable voter support.

Location 2(Day 2)

At location 2, since $195.16 < 261.59$, it follows that $K=0$ is an optimal campaign strategy for day 2 with associated accumulated campaign costs of 195.16 US\$ for the case of favorable voter support. Since $182.98 < 269.67$, it follows that $K=0$ is an optimal campaign strategy for day 2 with associated accumulated campaign costs of 182.98 US\$ for the case of unfavorable voter support.

VI. CONCLUSIONS AND DISCUSSION

An optimization model for campaign optimization strategies under voter support uncertainty was presented in this paper. The model determines an optimal campaign strategy and costs at campaign locations. The decision of whether or not to hold a political campaign is made using dynamic programming over a finite period planning horizon. Results from the model indicate optimal campaign strategies and costs for the given problem at each location. As a cost minimization method in political campaign strategies, computational efforts of using markov decision process approach provide promising results. However, further extensions of the research are vital to analyze the impact of nonstationary voter support on the campaign strategies. Special interest is also sought in further extending the model by considering campaign strategies for minimum costs in the context of Continuous Time Markov Chains (CTMC). As noted in the study, campaign cost comparisons were vital in determining the optimal campaign strategy for the two campaign locations. By the same token, classification of voter support as a two-state Markov chain facilitated modeling and optimization process at the chosen locations.

VII. REFERENCES

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